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Asymptotic Safety implies that observables including scattering amplitudes remain finite at the highest energy scales. Traditionally, this feature is connected to an interacting fixed point of the Wilsonian renormalization group that provides the high-energy completion of the theory. In this chapter, we discuss a different facet of Asymptotic Safety, reviewing its imprint on the quantum effective action. We start with a concise introduction to form factors in gravity and gravity-matter systems, before giving an overview of gravity-mediated scattering amplitudes derived from the quantum effective action. We illustrate the use of the framework based on the form factors appearing in the context of quadratic gravity and Asymptotic Safety, before making the connections to positivity bounds derived for low-energy effective field theories and the computation of form factors from first principles. We conclude that form factors offer a unique, unifying perspective on quantum gravity. In particular, they play a pivotal role in determining the phenomenological consequences of Asymptotic Safety at the level of observables.

渐近安全意味着包括散射振幅在内的可观测量在最高能标下仍保持有限。传统上，这一特性与威尔逊重整化群的相互作用不动点相关联，该不动点为理论提供了高能完备性。本章我们讨论渐近安全的不同面向，梳理它在量子有效作用量上留下的印记。我们首先简要介绍引力及引力-物质系统中的形状因子，随后综述从量子有效作用量导出的引力介导散射振幅。我们以二次引力与渐近安全语境下出现的形状因子为例，阐释了该框架的应用，之后建立起它与低能有效场论正性界以及从第一性原理计算形状因子的关联。我们的结论是，形状因子为量子引力提供了独特统一的视角。尤其值得注意的是，它们在确定渐近安全在可观测量层面的唯象后果中发挥着关键作用。

Keywords

关键词

Asymptotic Safety - Quantum effective action - Scattering amplitudes - Form factors · Functional renormalization group · Graviton propagator ·

渐近安全 - 量子有效作用量 - 散射振幅 - 形状因子 · 函数重整化群 · 引力子传播子 ·

Gravity-matter couplings

引力-物质耦合

Introduction

引言

The gravitational Asymptotic Safety program [1, 2] strives for a quantum theory of gravity valid on all scales. The construction is not limited to pure gravity and can be supplemented by a broad class of matter degrees of freedom [3]. As its characteristic feature, the program builds on well-established principles of relativistic quantum field theory (QFT) and postulates that the dynamics of gravity in the ultraviolet (UV) is controlled by an interacting renormalization group (RG) fixed point [4, 5] . In contrast to theories enjoying asymptotic freedom, interactions are not turned off in this regime though. Instead, classical and quantum contributions balance in a delicate way such that the high-energy regime of the theory enjoys an additional symmetry, the so-called quantum scale invariance [6]. It is expected that this feature entails the absence of unphysical UV divergences in physical observables. The ambition of the form factor framework to asymptotically safe quantum gravity, initiated in [7], is to make this expectation precise both at the level of scattering amplitudes and at the level of quantum corrections to spacetimes exhibiting singularities at the classical level.

引力渐近安全项目 [1, 2] 旨在建立一个适用于所有尺度的引力量子理论。该构造不限于纯引力，还可以涵盖广泛种类的物质自由度 [3]。该项目的核心特征是建立在相对论量子场论 (QFT) 已确立的原理之上，并假设引力在紫外 (UV) 的动力学由一个相互作用型重整化群 (RG) 不动点 [4, 5] 控制。与渐近自由理论不同，该情形下相互作用并不会消失。相反，经典贡献与量子贡献以一种微妙的方式平衡，使得该理论的高能区具备一种额外对称性，即所谓的量子标度不变性 [6]。一般认为这一特性会保证物理可观测量中不存在非物理的紫外发散。形式因子框架下渐近安全量子引力研究起源于文献 [7]，其目标是在散射振幅层面，以及经典层面存在奇点的时空的量子修正层面，将这一猜想严格化。

The pivotal element in the form factor framework is the quantum effective action Γ . By definition, the propagators and vertices derived from this generating functional are exact in the sense that they include all quantum corrections. Thus, quantum-corrected spacetimes may be constructed by solving the quantum equations of motion derived from Γ . Moreover, quantum-corrected scattering processes are described by tree-level Feynman diagrams built from the effective propagators and vertices provided by the quantum effective action. Thus the computation of Γ is considered as equivalent to solving the theory. Clearly, obtaining the quantum effective action from a first principle computation is then a notoriously hard problem. This applies in particular in the context of gravity, where the long-range nature of the gravitational force introduces nonlocal interaction terms at the effective level.

形式因子框架的核心是量子有效作用量 Γ。根据定义，从该生成泛函导出的传播子和顶点包含所有量子修正，因此是精确的。由此，我们可以通过求解由 Γ 导出的量子运动方程构造量子修正后的时空。此外，量子修正后的散射过程由树级费曼图描述，这些费曼图由量子有效作用量给出的有效传播子和有效顶点构造而成。因此，计算 Γ 被认为等价于求解该理论。显然，从第一性原理出发得到量子有效作用量是一个公认的难题，这一点在引力研究中尤为突出: 引力的长程性质会在有效层面引入非局域相互作用项。

Given this rather intimidating perspective, it is useful to break the analysis of Γ into several parts. Firstly, one would like to have a conceptual understanding of the building blocks that are essential for implementing the concept of Asymptotic Safety at the level of the quantum effective action. This leads to the form factors introduced in section ”The Quantum Effective Action Including Form Factors”. The characteristic feature of the form factors is that they encode the full momentum dependence of propagators and vertices in the context of a general spacetime. This makes the form factors an indispensable element for encoding the quantum-gravitational dynamics.

考虑到这一研究难点，将 Γ 的分析拆解为多个部分是十分有益的。首先，我们需要从概念层面理解在量子有效作用量层面实现渐近安全概念所必需的基础构件，这就引出了本文“包含形式因子的量子有效作用量”一节介绍的形式因子。形式因子的核心特点是，它可以在任意时空背景下编码传播子和顶点的完整动量依赖关系，因此是刻画量子引力动力学必不可少的要素。

The next step seeks to understand the implications of form factors at the level of physical processes. A prime example is the scattering of particles, where the amplitudes encoding the probability of a process receive nontrivial contributions from the form factors. The general structure of these amplitudes can be understood systematically by identifying all terms in Γ that contribute to a given scattering process. For a particular process with finitely many external particles, this classification involves only a finite number of form factors. In particular, it is not necessary to determine the quantum effective action in full generality. Once this classification is completed, one derives the most general amplitude compatible with the existence of a quantum effective action. The results obtained along these lines are summarized in section ”Classifying Two-to-Two Scattering Processes” where we explain the role of form factors in gravity-mediated two-to-two scattering processes with external matter fields.

下一步我们需要理解形式因子在物理过程层面的影响。粒子散射是一个典型例子: 描述过程概率的散射振幅会得到形式因子带来的非平庸贡献。我们可以通过识别 Γ 中对给定散射过程有贡献的所有项，系统地梳理这些振幅的一般结构。对于一个含有有限个外粒子的特定过程，该分类仅涉及有限个形式因子，尤其不需要完全确定整个量子有效作用量。完成分类后，我们就可以推导出与量子有效作用量的存在性自洽的最一般振幅。本文“分类二对二散射过程”一节总结了沿这一思路得到的结果，解释了形式因子在有外物质场参与、引力介导的二对二散射过程中的作用。

Based on this parametric approach, it is then natural to ask about the conditions on the form factors leading to asymptotically safe scattering amplitudes. A proof of principle demonstrating that the form factors indeed provide sufficient room to implement this scenario is provided in section ”Asymptotically Safe Scattering Amplitudes”. The key insights from this specific example are that asymptotically safe amplitudes may be obtained without introducing new degrees of freedom (akin to string theory): The specific example tames the growth of the amplitude at high energy through a Regge-like structure of first-order poles situated at purely imaginary squared momenta. The model also demonstrates that Asymptotic Safety at the level of amplitudes requires a delicate interplay between the momentum dependence in the propagators and vertices. While this ”tuning” may appear to be ad hoc at first sight, it is natural to expect that these structures will actually be provided by the interacting RG fixed point underlying Asymptotic Safety.

> 基于这套参数化方法，一个很自然的问题是: 形式因子需要满足什么条件才能得到渐近安全的散射振幅?“渐近安全散射振幅”一节给出了原理性证明，表明形式因子确实为实现该场景提供了足够空间。从这个具体例子中得到的核心洞见是，我们不需要引入新的自由度就可以得到渐近安全振幅(这点和弦论不同): 该具体例子通过一阶极点的 Regge 型结构抑制了振幅在高能区的增长，这些极点位于纯虚数平方动量处。该模型还表明，振幅层面的渐近安全要求传播子和顶点的动量依赖之间存在精细的相互作用。虽然这种“调谐”初看是人为引入的，但我们有理由期待这些结构实际上会由渐近安全背后的相互作用 RG 不动点自然给出。

A key question within the Asymptotic Safety program is whether the dynamics resulting from the RG fixed point is compatible with the principles of causality, unitarity, and positivity formulated at the level of the S-matrix. Important cross-checks in this direction arise from extracting the low-energy effective field theory from Γ by expanding the interactions in inverse powers of a UV cutoff scale and truncating the expansion at a fixed order. Based on low-energy effective field theory considerations, one can then derive consistency conditions on the couplings appearing in this expansion. Prominent examples are the constraints on the graviton three-point vertex derived by Maldacena et al. [8] as well as restrictions on the signs of the matter self-interactions, see, e.g., [9-12]. Section ”Connection to Low-Energy Effective Field Theory” gives a brief overview on such consistency checks. The typical assumptions made in their derivation are of relevance in the context of Asymptotic Safety.

> 渐近安全项目中的一个核心问题是，重整化群不动点产生的动力学是否符合在 S 矩阵层面提出的因果性、幺正性和正定性原理。这一方向的重要交叉检验来自于: 从 Γ 中提取低能有效场论，方法是将相互作用按紫外截断标度的负幂次展开，并将展开截断在固定阶次。基于低能有效场论的考量，我们可以对该展开中出现的耦合常数给出相容性条件。典型例子包括马尔达西那等人 [8] 推导得到的引力子三点顶点约束，以及对物质自相互作用符号的限制，参见例如 [9-12]。“与低能有效场论的联系”一节简要概述了这类相容性检验。其推导过程中所作的典型假设，在渐近安全的研究语境中具有重要意义。

The relevance of the form factor framework for implementing Asymptotic Safety clearly warrants the derivation of these functions from first principle computations. Section ”Form Factors from First Principles” briefly covers the two strategies that have been employed in this context: solutions of the functional RG, foremost the Wetterich equation [13-15], and the reconstruction of form factors based on correlation functions obtained from Monte Carlo simulations. While this program is still in its infancy, it provided some interesting pointers to the presence of nonlocal form factors which could lead to phenomenologically interesting modifications of the gravitational dynamics on macroscopic scales [16].

> 形式因子框架对实现渐近安全的重要性，显然要求我们从第一性原理计算推导出这些函数。“第一性原理得到的形式因子”一节简要介绍了该研究方向已采用的两种策略: 泛函重整化群的解，最主要的是韦特里奇方程 [13-15]，以及基于蒙特卡洛模拟得到的关联函数重构形式因子。尽管该项目仍处于起步阶段，它已经为非局域形式因子的存在提供了一些有趣的线索，这类形式因子可能会在宏观尺度上对引力动力学产生唯象学上值得关注的修正 [16]。

The main focus of this chapter is on the role of form factors within the gravitational Asymptotic Safety program. As stressed in our conclusions, section ”Conclusions”, the application range of this framework is actually much broader. In particular, the form factors readily capture quantum corrections arising within the effective field theory of quantum gravity [17], nonlocal ghost-free gravity [18-21], and perturbatively renormalizable approaches to quantum gravity [22- 24]. Therefore, the form factor framework has the potential of providing a unifying perspective on quantum gravity and its phenomenological consequences.

本章的核心关注点是形式因子在引力渐近安全项目中的作用。正如我们在结论部分“结论”中所强调的，该框架的实际应用范围要广泛得多。具体而言，形式因子可以很方便地描述量子引力有效场论 [17]、无鬼非局域引力 [18-21] 以及量子引力的微扰可重整化方案 [22-24] 中产生的量子修正。因此，形式因子框架有潜力为量子引力及其唯象学结果提供一个统一的研究视角。

The Quantum Effective Action Including Form Factors

包含形状因子的量子有效作用量

Form factors are the key elements for constructing (gravity-mediated) scattering amplitudes that are well behaved at trans-Planckian energy scales. This section introduces the concept of form factors for the effective action Γ built from a Lorentzian spacetime metric $g_{\mu\nu}$, an Abelian gauge field A_μ , an uncharged scalar field ϕ , and Dirac fermions ψ . In particular, we present the results from the classification program [7, 25-28], constructing all interaction monomials which contribute to the two-to-two scattering of matter fields in a flat spacetime.

形状因子是构造在跨普朗克能标下表现良好的 (引力介导的) 散射振幅的关键要素。本节介绍由洛伦兹时空度规 $g_{\mu\nu}$ 、阿贝尔规范场 A_μ 、不带电标量场 ϕ 和狄拉克费米子 ψ 构造的有效作用量 Γ 的形状因子概念。我们特别展示了分类项目 [7, 25-28] 的研究结果: 该项目构造了所有对平坦时空中物质场二对二散射有贡献的相互作用单项式。

Setup

设置

Here we collect the conventions used in the description of the effective action. In particular, we present our notation for spacetime fields and curvature, and underlying symmetry assumptions. We also introduce the notation used to efficiently denote form factors.

我们在此汇总描述有效作用量时使用的约定。具体而言，我们介绍时空场和曲率的记号、基本的对称性假设，还引入了用于简洁表示形状因子的记号。

Conventions: Fields and Bianchi Identities

约定: 场与比安基恒等式

We work on a generic four-dimensional spacetime with metric $g_{\mu\nu}$ and signature $\{+,-,-,-\}$. The covariant derivative associated with this metric is denoted by ∇_{μ} , and we introduce the covariant d'Alembertian $\Box \equiv -g^{\mu\upsilon}\nabla_{\mu}\nabla_{\upsilon}$. In order to ease our notation, the contraction of spacetime indices will frequently be denoted by ".," e.g., $\Box = -\nabla \cdot \nabla$. We define the Riemann tensor as

我们研究一般四维时空，其度规为 $g_{\mu\nu}$ ， 号差为 $\{+,-,-,-\}$ 。与该度规关联的协变导数记为 ∇_{μ} ，我们引入协变达朗贝尔算符 $\Box \equiv -g^{\mu\upsilon}\nabla_{\mu}\nabla_{\upsilon}$ 。为简化记号，时空指标的缩并常记为“.”，例如 $\Box = -\nabla \cdot \nabla$ 。我们将黎曼张量定义为

$$R_{\mu\nu\rho}{}^{\lambda} \equiv \partial_{\nu}\Gamma^{\lambda}{}_{\mu\rho} - \partial_{\mu}\Gamma^{\lambda}{}_{\nu\rho} + \Gamma^{\sigma}{}_{\mu\rho}\Gamma^{\lambda}{}_{\nu\sigma} + \Gamma^{\sigma}{}_{\nu\rho}\Gamma^{\lambda}{}_{\mu\sigma}, \tag{1}$$

and the Ricci tensor and Ricci scalar are $R_{\mu\nu} = R_{\mu\lambda\nu}{}^{\lambda}$ and $R = g^{\mu\upsilon}R_{\mu\upsilon}$. The Riemann tensor satisfies the Bianchi identities

里奇张量和里奇标量分别为 $R_{\mu\nu} = R_{\mu\lambda\nu}{}^{\lambda}$ 和 $R = g^{\mu\upsilon}R_{\mu\upsilon}$ 。黎曼张量满足比安基恒等式

$$R_{\mu[\upsilon\rho\sigma]} = 0,\ \nabla_{[\alpha}R_{\mu\upsilon]\rho\sigma} = 0. \tag{2}$$

Here $[\cdots]$ denotes anti-symmetrization with unit strength. The second Bianchi identity implies the contracted Bianchi identities

此处 $[\cdots]$ 表示单位强度反对称化。第二比安基恒等式可推导出缩并比安基恒等式

$$\nabla^{\alpha}R_{\alpha\beta\mu\upsilon} = 2\nabla_{[\mu}R_{\upsilon]\beta},\ \nabla^{\upsilon}R_{\mu\upsilon} = \frac{1}{2}\nabla_{\mu}R. \tag{3}$$

As a direct consequence, one has that

由此可直接得到

$$\Box R_{\rho\sigma\mu\nu} = 2\nabla_{\sigma}\nabla_{[\mu}R_{\nu]\rho} - 2\nabla_{\rho}\nabla_{[\mu}R_{\nu]\sigma} + O\left(R^{2}\right). \tag{4}$$

Contracting the open indices with the Riemann tensor and integrating by parts then yield the relation [7, 29, 30]

将自由指标与黎曼张量缩并再分部积分，可得到关系 [7, 29, 30]

$$\int \mathrm{d}^{4}x\sqrt{-g}\left\{R^{\rho\sigma\mu\upsilon}\Box^{n}R_{\rho\sigma\mu\upsilon} - 4R^{\mu\upsilon}\Box^{n}R_{\mu\nu} + R\Box^{n}R\right\} = O\left(R^{3}\right),\ n \geq 1,$$

(5)

where the right-hand side denotes terms which are of third order in the spacetime curvature tensors. In four dimensions, this identity is complemented by the Gauss-Bonnet identity, stating that the combination

其中右侧表示时空曲率张量的三阶项。在四维中，该恒等式可由高斯-博内恒等式补充，该恒等式指出如下组合

$$E = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \tag{6}$$

is topological in the sense that it does not contribute to the equations of motion. Finally, it is convenient to express the Riemann tensor in terms of the Weyl tensor

是拓扑项，即它不会对运动方程产生贡献。最后，将黎曼张量用外尔张量表示会更方便

$$C_{\mu\upsilon\rho\sigma} = R_{\mu\upsilon\rho\sigma} - g_{\mu[\rho}R_{\sigma]\upsilon} + g_{\upsilon[\rho}R_{\sigma]\mu} + \frac{1}{3}Rg_{\mu[\rho}g_{\sigma]\upsilon}. \tag{7}$$

In the gauge sector, we introduce the gauge-invariant Abelian field strength tensor of the photon A_μ ,

在规范 sector 中，我们引入光子的规范不变阿贝尔场强张量 A_μ ，

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{8}$$

The field strength tensor satisfies a Bianchi identity

场强张量满足比安基恒等式

$$\nabla_{[\alpha}F_{\beta\gamma]} = 0. \tag{9}$$

In the fermionic sector, we introduce covariant Dirac matrices $\{\gamma_\mu\}$ that satisfy the anti-commutator

在费米子 sector 中，我们引入满足如下对易关系的协变狄拉克矩阵 $\{\gamma_\mu\}$

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}\mathbb{1}. \tag{10}$$

This gives the Dirac operator $\nabla\!\!\!/ = g^{\mu\upsilon}\gamma_\mu\nabla_\upsilon$, which satisfies the Lichnerowicz relation:

由此得到狄拉克算符 $\nabla\!\!\!/ = g^{\mu\upsilon}\gamma_\mu\nabla_\upsilon$ ，它满足利奇诺维奇关系:

$$\Delta_{\mathrm{D}} \equiv (\mathrm{i}\nabla\!\!\!/)^2 = \left(\square + \frac{1}{4}R\right)\mathbb{1}. \tag{11}$$

Furthermore, we have the matrix

此外，我们定义如下矩阵

$$\gamma_\star = \frac{1}{24}\sqrt{-g}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\upsilon\gamma_\rho\gamma_\sigma. \tag{12}$$

We use the symbols Φ and Ψ to denote generic fields which can be either metric fluctuations, gauge, or matter fields.

我们使用符号 Φ 和 Ψ 表示一般场，可以是度规涨落、规范场或物质场。

Form Factors

形状因子

The key idea of a form factor is to promote the coupling constants associated with a given interaction term to a momentum-dependent function. A prototypical example is provided by the electric charge e appearing in electrodynamics,

形状因子的核心思想是将给定相互作用项对应的耦合常数推广为依赖动量的函数。一个典型例子是电动力学中出现的电荷 e ,

$$\frac{1}{4}\int \mathrm{d}^4x\sqrt{-g}F_{\mu\nu}\frac{1}{e^2}F^{\mu\nu}\mapsto\frac{1}{4}\int \mathrm{d}^4x\sqrt{-g}F_{\mu\nu}\frac{1}{e^2\left(\Box\right)}F^{\mu\nu}, \tag{13}$$

where e^2 is promoted to a function $e^2\left(\Box\right)$. In this way the (corrections to) dynamics is encoded in a manifestly background-independent way. In Minkowski spacetime, the $e^2\left(\Box\right)$ reduces to a momentum-dependent function via Fourier transformation. In the case of curved spacetime, this is straightforwardly generalized by replacing partial derivatives by covariant ones. In this way, the form factors capture the wellknown momentum dependence (colloquially also called the running) of couplings observed in particle physics experiments [31] at the level of the effective action. The form factor framework implements this generalization in a systematic way.

其中 e^2 被推广为函数 $e^2\left(\Box\right)$ 。通过这种方式，动力学 (及其修正) 以明显背景无关的方式被编码。在闵氏时空中，通过傅里叶变换，$e^2\left(\Box\right)$ 可约化为依赖动量的函数。在弯曲时空背景下，只需将偏导数替换为协变导数即可直接推广这一构造。通过这种方式，形状因子在有效作用量层面体现了粒子物理实验观测到的耦合常数著名的动量依赖 (俗称跑动)，形状因子框架以系统化的方式实现了这一推广。

Generically, the form factors depend on all independent contractions of the covariant derivatives. By partial integration, we can reduce the number of arguments of the form factors. We adopt the following notation: For a form factor acting on $\Phi_1\cdots\Phi_n$, the arguments are written as

一般来说，形状因子依赖于协变导数所有独立缩并。通过分部积分，我们可以减少形状因子自变量的数量。我们采用如下记号: 对于作用在 $\Phi_1\cdots\Phi_n$ 上的形状因子，自变量记为

$$\int \mathrm{d}^4x\sqrt{-g}\Phi_1 f\left(\Box\right)\Phi_2,\quad \int \mathrm{d}^4x\sqrt{-g}f\left(\Box_1,\Box_2,\Box_3\right)\Phi_1\Phi_2\Phi_3, \tag{14}$$

and

和

$$\int \mathrm{d}^4x\sqrt{-g}f\left(\left\{-\nabla_i\cdot\nabla_j\right\}_{1\le i<j\le n}\right)\Phi_1\cdots\Phi_n,\ n\ge 4. \tag{15}$$

In particular, form factors associated with monomials containing four fields generically have six independent arguments. The subscript on each operator denotes the field it acts on, for example, $\nabla_1\left(\Phi_1\Phi_2\right)=\left(\nabla_1\Phi_1\right)\Phi_2$. In order to ease the notation, we will often suppress the arguments of the form factors.

特别地，与含四个场的单项式对应的形状因子一般有六个独立自变量。每个算符的下标标识它作用的场，例如 $\nabla_1(\Phi_1\Phi_2) = (\nabla_1\Phi_1)\Phi_2$ 。为简化记号，我们通常会省略形状因子的自变量。

Symmetry Assumptions

对称性假设

We constrain the terms tracked in our classification by imposing the following symmetry requirements. The gauge field A_μ comes with a U(1) gauge symmetry. Therefore, the dependence of Γ on the photon must be in the form of the field strength tensor (8). We will assume that both scalars and fermions are uncharged under U(1). Furthermore, we impose that each scalar field comes with a global $\mathbb{Z}_2$ -symmetry, so that only even powers appear in the effective action. In addition, the complete action is invariant under diffeomorphism symmetry.

我们通过施加以下对称性要求来约束分类中追踪的项。规范场 A_μ 具有 U(1) 规范对称性。因此，Γ 对光子的依赖必须以场强张量 (8) 的形式呈现。我们假设标量和费米子在 U(1) 下均不带电。此外，我们要求每个标量场都带有整体 $\mathbb{Z}_2$ 对称性，因此有效作用量中仅出现偶次幂。另外，完整作用量在微分同胚对称性下保持不变。

Classifying the Interactions Within the Effective Action

对有效作用量内的相互作用进行分类

We are interested in finding all interaction monomials that can contribute to the graviton-mediated two-to-two particle scattering process. Since the n -point function is obtained by taking n functional derivatives, it is in general determined by terms in Γ containing n fields. This motivates the notation of $\Gamma_{\Phi^n\Psi^m}$ to denote the building block of Γ containing n fields of type Φ and m fields of type Ψ .

我们旨在找出所有可贡献于引力子介导的二对二粒子散射过程的相互作用单项式。由于 n 点函数是通过取 n 次泛函导数得到的，它通常由 Γ 中包含 n 个场的项决定。因此我们引入记号 $\Gamma_{\Phi^n\Psi^m}$ ，用来表示 Γ 的构造块，其中包含 n 个 Φ 类型场和 m 个 Ψ 类型场。

Gravitons (and non-Abelian gauge fields) form a notable exception to this classification. Since each term in the effective action will contain a factor $\sqrt{-g}$, any term in Γ will be nonlinearly coupled to gravity. Moreover, spacetime curvature tensors contain infinitely many powers of the metric fluctuations. However, since we are expanding around a flat background, any term containing more than n curvature tensors will not contribute to a vertex with n graviton legs. Therefore, we adopt the convention that Γ_{h^n} contains up to n curvature tensors, while for $m \geq 1$, the building block $\Gamma_{h^n\Phi^m}$ contains exactly n curvature tensors and m fields of type Φ .

引力子 (以及非阿贝尔规范场) 是该分类的一个明显例外。由于有效作用量中的每一项都包含因子 $\sqrt{-g}$ ，因此 Γ 中的任意项都与引力非线性耦合。此外，时空曲率张量包含无穷多阶度量涨落幂次。但由于我们是在平直背景附近展开，任何包含超过 n 个曲率张量的项都不会对带有 n 个引力子外腿的顶点产生贡献。因此我们约定: Γ_{h^n} 最多包含 n 个曲率张量，而对于 $m \geq 1$ ，构造块 $\Gamma_{h^n\Phi^m}$ 恰好包含 n 个曲率张量和 m 个 Φ 类型场。

Pure Gravity

纯引力

Let us start by discussing the gravitational sector. Thinking about gravitons $h_{\mu\upsilon}$ propagating in a flat Minkowski metric $\eta_{\mu\upsilon}$, it is natural to organize the effective action in terms of an expansion in powers of the spacetime curvature. Neglecting terms that are cubic in the Riemann tensor or its contractions, the most general form of the effective action is given by

我们先从讨论引力 sector 开始。对于在平直闵氏度规 $\eta_{\mu\upsilon}$ 中传播的引力子 $h_{\mu\upsilon}$ ，很自然会将有效作用量按时空曲率的幂次展开整理。忽略黎曼张量或其缩并中三次方的项后，有效作用量最一般的形式为

$$\Gamma_{h^2} = \frac{1}{16\pi G_N} \int \mathrm{d}^4x\sqrt{-g}\left[2\Lambda - R - \frac{1}{6}R f_{RR}(\Box) R + \frac{1}{2}C_{\mu\upsilon\rho\sigma} f_{CC}(\Box) C^{\mu\upsilon\rho\sigma}\right].$$

(16)

Here G_N is Newton’s constant, Λ denotes the cosmological constant, and $f_{RR}(\Box)$ and $f_{CC}(\Box)$ are the two form factors appearing at quadratic order in the spacetime curvature. We stress that these couplings are effective in the sense that they incorporate all quantum corrections. This also entails that G_N and Λ are constant [7,32]. Similarly, (16) entails that the couplings appearing in the quadratic part of the action can develop a momentum dependence which is encoded in the corresponding form factors.

此处 G_N 是牛顿常数，Λ 代表宇宙学常数，$f_{RR}(\Box)$ 和 $f_{CC}(\Box)$ 是时空曲率二次阶出现的两个形状因子。我们需要强调，这些耦合都是有效耦合: 它们已经包含了所有量子修正。这也意味着 G_N 和 Λ 是常数 [7,32]。同理，式 (16) 表明，作用量二次部分的耦合可以依赖动量，这种依赖就编码在对应的形状因子中。

The expansion (16) is complete in the sense that there is no contribution associated with the square of the Ricci tensor. Any contributions containing the d’Alembertian can be eliminated via the identity (5), while the pure $R_{\mu\upsilon}R^{\mu\upsilon}$ -term can be rewritten in terms of the Gauss-Bonnet combination (6) and then does not enter physical processes. When writing (16), we adopted the ”Weyl basis” which leads to an easy relation between the form factors f_{RR}, f_{CC} and the graviton propagator. The identities (7) and (5) allow to map this result to the Ricci basis where the form factors are associated with the squares of the Ricci scalar and Ricci tensor.

展开式 (16) 是完备的: 里奇张量的平方不存在额外贡献。任何包含达朗贝尔算符的贡献都可以通过恒等式 (5) 消去，而纯 $R_{\mu\upsilon}R^{\mu\upsilon}$ 项可以通过高斯-博内组合 (6) 改写，因此不会进入物理过程。我们在写出式 (16) 时采用了“外尔基”，它能方便地建立形状因子 f_{RR}, f_{CC} 和引力子传播子之间的关系。利用恒等式 (7) 和 (5)，可以将我们的结果映射到里奇基，在里奇基中形状因子对应里奇标量平方和里奇张量平方。

While (16) suffices to determine the graviton propagator in flat space, studying two-to-two graviton scattering requires extending this result to quartic order in the spacetime curvature. Building on the FKWC classification [33], a series of geometric identities valid at cubic order has been published in [34]. A local basis for invariants at cubic order in the spacetime curvature can be found in [2], and an extension to fourth order, tailored to graviton scattering, has been pursued in [35], see also [36] for a basis given in the context of the functional RG. Owed to the Bianchi identity, the generalization of these results including form factors is a formidable task. Some systematics in a nonlocal basis have been developed in the context of heat kernel computations [37, 38] .

虽然式 (16) 足以确定平直空间中的引力子传播子，但研究双引力子散射需要将这个结果推广到时空曲率的四阶。基于 FKWC 分类 [33]，文献 [34] 发表了一系列三次阶成立的几何恒等式。文献 [2] 给出了时空曲率三次阶不变量的一组局域基，文献 [35] 完成了适用于引力子散射的四阶推广，关于泛函重整化群背景下的基可见文献 [36]。由于比安基恒等式，将这些结果推广到包含形状因子的情况是一项艰巨的任务。目前在热核计算 [37, 38] 的背景下，已经发展出了非局域基的部分系统方法。

Gravity Coupled to Scalar Matter

引力与标量物质耦合

We proceed by considering gravity coupled to an uncharged, massive real scalar field ϕ . Following the example (13), the kinetic term including the form factor is

我们接下来研究引力与不带电的有质量实标量场 ϕ 耦合。参考例 (13)，包含形状因子的动能项为

$$\Gamma_{\phi^2} = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\phi f_{\phi\phi}(\Box)\,\phi. \tag{17}$$

The form factor is normalized such that $f'_{\phi\phi}\left(m^2\right) = 1$, which ensures that the scalar field is canonically normalized on-shell.

形状因子按 $f'_{\phi\phi}\left(m^2\right) = 1$ 归一化，这保证了标量场在在壳时满足正则归一化。

The construction of a basis for the interaction vertices has to account for the redundancies due to partial integration. In flat space, this amounts to rewriting the momentum dependence of the vertices by exploiting momentum conservation. A basis for the graviton-scalar-scalar vertex is then provided by

构建相互作用顶点的基时，必须考虑分部积分带来的冗余性。在平坦空间中，这相当于利用动量守恒改写顶点的动量依赖关系。引力子-标量-标量顶点的一组基由下式给出

$$\Gamma_{h\phi^2} = \int d^4x\sqrt{-g}\left[f_{R\phi\phi}R\phi\phi + f_{\mathrm{Ric}\,\phi\phi}R^{\mu\nu}\left(\nabla_\mu\phi\right)\left(\nabla_\nu\phi\right)\right]. \tag{18}$$

The four-point vertex for the scalar self-interaction is created by a single term,

标量自相互作用的四点顶点仅由一项构成,

$$\Gamma_{\phi^4} = \int d^4x\sqrt{-g}f_{\phi^4}\phi\phi\phi\phi. \tag{19}$$

In the latter case, all derivatives acting on the scalars are created through the form factor. This is readily seen by noting that the six independent arguments in f_{ϕ^4} suffice to generate all possible contracted derivatives acting on the three left-most ϕ fields. At the same time any derivative acting on the fourth scalar field can always be removed by partial integration.

在后一种情况中，所有作用在标量场上的导数都由形状因子产生。不难发现，f_{ϕ^4} 中的六个独立自变量足以生成所有作用在最左侧三个 ϕ 场上的缩并导数，同时作用在第四个标量场上的任意导数都总能通过分部积分消去。

At this stage it is instructive to provide an explicit example on how non-basis monomials are mapped to basis elements. For explicitness, we consider a contribution to the $h\phi\phi$ -vertex of the form

在此阶段，给出一个非基单项式如何映射到基元的显式示例很有启发意义。为清晰起见，我们考虑 $h\phi\phi$ 顶点的一个如下形式的贡献

$$I = \int d^4x\sqrt{-g}f_{R\phi\phi}\left(\Box_1,\Box_2,\Box_3\right)R\left(\nabla_\mu\phi\right)\left(\nabla^\mu\phi\right). \tag{20}$$

The symmetry in the scalar fields ensures that $f_{R\phi\phi}$ is symmetric in its last two arguments. In order to map I to the basis (18), we rewrite the term as

标量场的对称性保证 $f_{R\phi\phi}$ 对最后两个自变量对称。为了将 I 映射到基 (18)，我们把该项改写为

$$\begin{aligned} I &= -\frac{1}{2}\int d^4x\sqrt{-g}f_{R\phi\phi}\left(\Box_1,\Box_2,\Box_3\right)R\left[\Box\left(\phi\phi\right)-\left(\Box\phi\right)\phi-\phi\left(\Box\phi\right)\right] \\ &= -\frac{1}{2}\int d^4x\sqrt{-g}f_{R\phi\phi}\left(\Box_1,\Box_2,\Box_3\right)\left[\Box_1-\Box_2-\Box_3\right]R\phi\phi. \end{aligned} \tag{21}$$

The last line matches the structure of the first term in (18). Thus at this point (20) has been mapped to the basis provided in (18).

最后一行与 (18) 中第一项的结构匹配，因此此时 (20) 已经被映射到 (18) 给出的基中。

Gravity Coupled to Photons

引力与光子耦合

The classification of interactions among gravitons and photons follows along similar lines as the scalar case. Introducing a form factor, the photon kinetic term becomes

引力子与光子之间的相互作用分类遵循与标量场情况类似的思路。引入形状因子后，光子动能项变为

$$\Gamma_{A^2} = -\frac{1}{4}\int \mathrm{d}^4x\sqrt{-g}F_{\mu\nu}f_{FF}\left(\Box\right)F^{\mu\nu}. \tag{22}$$

The overall normalization of $f_{FF}\left(\Box\right)$ is again fixed by imposing that the field is canonically normalized on-shell: $f_{FF}\left(0\right) = 1$.

$f_{FF}\left(\Box\right)$ 的整体归一化同样由壳上场的正则归一化条件确定: $f_{FF}\left(0\right) = 1$ 。

The construction of a basis for the interaction vertices including the appropriate form factors is slightly more involved than in the scalar case, since the photon case has three operations which allow us to map interaction monomials to each other. Firstly, there is partial integration. Secondly, one has to account for the antisymmetry of the field strength $F_{\mu\nu}$. Thirdly, derivatives acting on the field strength can be rewritten by applying the Bianchi identity (9). Following [28], a basis can be identified based on the following algorithm: One starts with the product of a fixed number of uncontracted field strength tensors and covariant derivatives. Subsequently, one constructs the highly over-complete set of interaction monomials by performing all possible contractions of the spacetime indices. Finally, this set is reduced to the minimal number of independent terms by applying all symmetry operations specified above in a systematic way.

构造包含合适形状因子的相互作用顶点基比标量场情况稍复杂, 因为光子情况存在三种可将相互作用单项式相互映射的操作: 第一是分部积分；第二需要考虑场强 $F_{\mu\nu}$ 的反对称性；第三，作用在场强上的导数可以通过应用比安基恒等式 (9) 改写。遵循文献 [28] 的方法，我们可以基于以下算法确定一组基: 首先从固定数量的未缩并场强张量与协变导数的乘积出发，随后通过对时空指标做所有可能的缩并，构造出高度过完备的相互作用单项式集合，最后通过系统应用上述所有对称操作，将该集合约化为最小数量的独立项。

For the graviton-photon-photon vertex extracted from interaction monomials with one spacetime curvature, this procedure identifies seven independent form factors. These can be chosen according to

对于从含一个时空曲率的相互作用单项式中提取出的引力子-光子-光子顶点，该流程确定了七个独立形状因子，可按如下方式选取

$$\begin{aligned}\Gamma_{hA^2} = \int \mathrm{d}^4x\sqrt{-g}\,\big[& f_{RFF}RF_{\alpha\beta}F^{\alpha\beta} + f_{\mathrm{Ric}FF}R^{\alpha\beta}F_{\alpha}{}^{\gamma}F_{\beta\gamma} \\ & + f_{\mathrm{Rm}FF}R_{\alpha\beta\gamma\delta}F^{\alpha\beta}F^{\gamma\delta} \\ & + f_{D^2RFF}\left(\nabla^{\alpha}\nabla^{\beta}R\right)F_{\alpha}{}^{\gamma}F_{\beta\gamma} + f_{D^2\,\mathrm{Ric}\,FF}\left(\nabla^{\alpha}\nabla^{\beta}R^{\gamma\delta}\right)F_{\alpha\gamma}F_{\beta\delta} \\ & + f_{\mathrm{Ric}D^2FF}R^{\gamma\delta}\left(\nabla^{\alpha}\nabla^{\beta}F_{\alpha\gamma}\right)F_{\beta\delta} + f_{\mathrm{Ric}DFDF}R^{\gamma\delta}\left(\nabla^{\alpha}F_{\alpha\gamma}\right)\left(\nabla^{\beta}F_{\beta\delta}\right)\big].\end{aligned}$$

(23)

The identity (4) then leads to constraints on the functional form of the form factors. For instance $f_{\mathrm{Rm}FF}$ is independent of $\Box_1$ since the Bianchi identity satisfied by the Riemann tensor allows to map terms of the form $\Box R_{\rho\sigma\mu\nu}$ to other basis elements in (23). In a nonlocal basis, this term can be removed entirely.

恒等式 (4) 会对形状因子的函数形式给出约束。例如 $f_{\mathrm{Rm}FF}$ 独立于 $\Box_1$ ，因为黎曼张量满足的比安基恒等式可将 $\Box R_{\rho\sigma\mu\nu}$ 形式的项映射到 (23) 中的其他基元，在非局部基中，该项可以被完全移除。

Similarly, one concludes that the four-photon self-interactions contain seven free functions that can be chosen according to

同理可得，四光子自相互作用包含七个自由函数，可按如下方式选取

$$\begin{aligned}\Gamma_{A^4} = \int \mathrm{d}^4x\sqrt{-g}\,\big[& f_{F^2F^2}F_{\alpha\beta}F^{\alpha\beta}F_{\gamma\delta}F^{\gamma\delta} + f_{F^4}F_\alpha{}^\beta F_\beta{}^\gamma F_\gamma{}^\delta F_\delta{}^\alpha \\ & + f_{FFDFDF_1}F_\alpha{}^\gamma F^{\delta\zeta}\left(\nabla^\alpha F_{\beta\delta}\right)\left(\nabla^\beta F_{\gamma\zeta}\right) \\ & + f_{FFDFDF_2}F_\beta{}^\gamma F^{\delta\zeta}\left(\nabla^\alpha F_{\alpha\gamma}\right)\left(\nabla^\beta F_{\delta\zeta}\right) \\ & + f_{FFDFDF_3}F_{\alpha\beta}F^{\gamma\delta}\left(\nabla^\alpha F_\gamma{}^\zeta\right)\left(\nabla^\beta F_{\delta\zeta}\right) \\ & + f_{FFDFDF_4}F_\beta{}^\gamma F_{\alpha\gamma}\left(\nabla^\alpha F^{\delta\zeta}\right)\left(\nabla^\beta F_{\delta\zeta}\right) \\ & + f_{FFD^2FD^2F}F_{\alpha\gamma}F_{\beta\delta}\left(\nabla^\alpha\nabla^\beta F^{\zeta\kappa}\right)\left(\nabla^\gamma\nabla^\delta F_{\zeta\kappa}\right)\big]\,. \end{aligned} \tag{24}$$

We note that the first line can also be formulated in terms of the dual field strength $\widetilde{F}_{\mu\nu} \equiv \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$. Applying standard identities for products of totally antisymmetric tensors, one has

我们注意到，第一行也可以用对偶场强 $\widetilde{F}_{\mu\nu} \equiv \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$ 表示。应用全反对称张量乘积的标准恒等式，可得

$$2\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + \left(F_{\mu\nu}\widetilde{F}^{\mu\nu}\right)^2 = 4F_\alpha{}^\beta F_\beta{}^\gamma F_\gamma{}^\delta F_\delta{}^\alpha. \tag{25}$$

This relates the basis used in (24) to the standard formulation of the Euler-Heisenberg-Lagrangian.

这将 (24) 中使用的基与欧拉-海森堡拉格朗日量的标准形式联系了起来。

Remarkably, the inclusion of form factors also leads to new interactions that otherwise would vanish due to the anti-symmetry of the field strength. In particular, there are two form factors associated with the three-photon vertex

值得注意的是，引入形状因子还会产生新的相互作用，若不引入形状因子，这些相互作用会因场强的反对称性而消失。具体来说，三光子顶点对应两个形状因子

$$\Gamma_{A^3} = \int d^4x\sqrt{-g}\left[f_{F^3}F_\alpha{}^\beta F_\beta{}^\gamma F_\gamma{}^\alpha + f_{FDFDF}F^{\gamma\delta}\left(\nabla^\alpha F_{\alpha\gamma}\right)\left(\nabla^\beta F_{\beta\delta}\right)\right].$$

(26)

We stress that while one can clearly write down these terms as part of the classification program, this does not necessarily entail that the corresponding form factors will actually appear in the effective action. Their appearance can be obstructed by global symmetries as, e.g., invariance of the action under $F_{\mu\nu} \mapsto -F_{\mu\nu}$ which is respected by (23) and (24) but broken in (26).

我们要强调，尽管我们显然可以在分类框架下写出这些项，但这不意味着对应的形状因子一定会出现在有效作用量中。它们的出现会被整体对称性阻碍，例如，作用量在 $F_{\mu\nu} \mapsto -F_{\mu\nu}$ 下的不变性被 (23) 和 (24) 满足，但在 (26) 中会被破坏。

Matter Self-interactions Coupling Scalars and Photons

耦合标量场与光子的物质自相互作用

As a side product of the classification described in the previous subsection, one can also determine a basis for scalar-photon interactions, including form factors. For two scalars and one photon, we have

作为上一小节分类工作的副产物，我们还可以确定包含形状因子的标量-光子相互作用的一组基。对于两个标量和一个光子，我们得到

$$\Gamma_{A\phi^2} = \int d^4x\sqrt{-g}f_{F\phi^2}F^{\alpha\beta}\left(\nabla_\alpha\phi\right)\left(\nabla_\beta\phi\right), \tag{27}$$

where $f_{F\phi^2}(\Box_1,\Box_2,\Box_3)$ must be anti-symmetric in the last two arguments to account for the anti-symmetry of the field strength tensor. Finally, the two-scalar-two-photon vertex is fixed by

其中 $f_{F\phi^2}(\Box_1,\Box_2,\Box_3)$ 关于最后两个自变量必须是反对称的，以满足场强张量的反对称性要求。最后，双标量双光子顶点由下式确定

$$\Gamma_{A^2\phi^2} = \int d^4x\sqrt{-g}\left[f_{FF\phi^2}F_{\alpha\beta}F^{\alpha\beta}\phi\phi + f_{FFD\phi D\phi}F_\alpha{}^\gamma F_{\gamma\beta}\left(\nabla^\alpha\phi\right)\left(\nabla^\beta\phi\right)\right.$$

$$\left. + f_{FFD^2\phi\phi}F_\alpha{}^\gamma F_{\gamma\beta}\left(\nabla^\alpha\nabla^\beta\phi\right)\phi + f_{FFD^2\phi D^2\phi}F_{\alpha\gamma}F_{\beta\delta}\left(\nabla^\alpha\nabla^\beta\phi\right)\left(\nabla^\gamma\nabla^\delta\phi\right)\right]$$

(28)

Fermions

费米子

The form factor framework is readily extended to Dirac spinors ψ , but it has not been carried out yet to a similar degree as for the other nongravitational fields. In this context, we first note that the Dirac operator satisfies the Lichnerowicz relation (11). Thus, it is natural to choose the differential operator appearing as the argument of the form factor as $\Delta_{\mathrm{D}} \equiv (\mathrm{i}\nabla)^2$. This entails that the form factors actually commute with the Dirac operator. In a similar fashion, one would use the gauge-covariant derivative for fields charged under a gauge symmetry.

形状因子框架可以很容易地推广到狄拉克旋量 ψ ，但目前尚未达到与其他非引力场相近的完善程度。在此我们首先指出，狄拉克算符满足里奇诺维茨关系 (11)。因此，将形状因子的微分算子自变量选为 $\Delta_{\mathrm{D}} \equiv (\mathrm{i}\nabla)^2$ 是自然的，这意味着形状因子实际上与狄拉克算符对易。类似地，对于规范对称下带电的场可以使用规范协变导数。

For uncharged fermions and up to quadratic order in ψ , we then encounter four form factors, one associated with the kinetic term and mass term of each chirality. Thus we can form the following bilinears [7]:

对于不带电的费米子，在 ψ 的二阶范围内我们总共得到四个形状因子，每个手征的动能项和质量项各对应一个形状因子，因此我们可以构造如下双线性项 [7]:

$$\Gamma_{\overline{\psi}\psi} = \int \mathrm{d}^4x\sqrt{-g}\,\overline{\psi}\left(f_{\overline{\psi}\psi,1}(\Delta_{\mathrm{D}})\left(\mathrm{i}\nabla\!\!\!/\right) + f_{\overline{\psi}\psi,2}(\Delta_{\mathrm{D}})\gamma_{\star}\left(\mathrm{i}\nabla\!\!\!/\right) \right. \tag{29}$$

$$\left. + f_{\overline{\psi}\psi,3}(\Delta_{\mathrm{D}})\mathbb{1} + f_{\overline{\psi}\psi,4}(\Delta_{\mathrm{D}})\gamma_{\star}\right)\psi\,.$$

This concludes our classification of interaction monomials which can appear in the effective action. As we will see in section ”Classifying Two-to-Two Scattering Processes”, the results are sufficient to describe the most general gravity-mediated two-to-two scattering amplitudes including scalars and photons as external particles.

至此我们完成了有效作用量中所有可能出现的相互作用单项式的分类。正如我们在“分类二对二散射过程”一节中将会看到的，上述结果足以描述最一般的引力介导二对二散射振幅，其中包含了作为外粒子的标量和光子。

Field Redefinitions and Inessential Operators

场重新定义与非本质算符

We conclude our discussion of parameterizations of the effective action with two important technical remarks. Starting from the scalar kinetic term (17), it is tempting to write

我们通过两条重要的技术说明，结束对有效作用量参数化的讨论。从标量动能项 (17) 出发，我们很容易会写出

$$f_{\phi\phi}(\square) = \left(\square - m^2\right)\widetilde{f}_{\phi\phi}(\square) \tag{30}$$

and to subsequently absorb the factor $\widetilde{f}_{\phi\phi}(\square)$ in a momentum-dependent field redefinition

随后将因子 $\widetilde{f}_{\phi\phi}(\Box)$ 吸收进依赖动量的场重新定义中

$$\phi \mapsto \widetilde{\phi} \equiv \left(\widetilde{f}_{\phi\phi}(\Box)\right)^{1/2} \phi. \tag{31}$$

This would remove the form factor from the kinetic term at the expense of modifying the form factors encoding the momentum-dependent interaction. Clearly, physics should not be affected by this redefinition. The decomposition (30) followed by (31) presupposes that we have identified the degrees of freedom of the theory (encoded in the poles of the scalar propagator) to be the ones of a massive scalar field with mass m . In this case, $\widetilde{f}_{\phi\phi}(\Box)$ will be a positive and invertible function and the field redefinition is well defined. This logic fails, however, if the theory contains additional degrees of freedom inducing zeros in $\widetilde{f}_{\phi\phi}(\Box)$. In this case, the field redefinition (31) would be ill-defined. Heuristically, this is easily understood from the observation that the theories before and after the field redefinition would differ in their degrees of freedom. In order to be as general as possible, we therefore retain the form factors in the kinetic terms in our classification.

这会移除动能项中的形状因子，但代价是修改了编码动量依赖相互作用的形状因子。显然，物理过程不会受这场重新定义的影响。分解式 (30) 后接 (31) 的前提是，我们已经将理论的自由度 (编码在标量传播子的极点中) 认定为质量为 m 的有质量标量场的自由度。在此情况下，$\widetilde{f}_{\phi\phi}(\Box)$ 会是正定可逆函数，场重新定义是良定义的。但若理论包含额外自由度，会导致 $\widetilde{f}_{\phi\phi}(\Box)$ 中出现零点，这一逻辑就不再成立。此时场重新定义 (31) 是不良定义的。从直观上很容易理解: 场重新定义前后的理论自由度不同，因此会出现该问题。为了尽可能保持一般性，我们在分类中保留了动能项里的形状因子。

More generally, our classification does not include information about interactions which can be removed by field redefinitions-called redundant (or inessential) operators [39-43]. By definition, a redundant operator does not contribute to observables. It is a scaling operator with respect to the RG which vanishes upon imposing the equations of motion. The second property implies that it can be written as

更一般地说，我们的分类不包含可通过场重新定义移除的相互作用的相关信息——这类相互作用被称为冗余 (或非本质) 算符 [39-43]。根据定义，冗余算符不对可观测量产生贡献。它是 RG 下的标度算符，在施加运动方程后等于零。第二条性质意味着它可以写为

$$\mathcal{O} = \int d^4x \sqrt{-g} \left[\frac{1}{\sqrt{-g}} \frac{\delta\Gamma}{\delta\Phi} \right] \cdot F[\Phi]. \tag{32}$$

The functional $F[\Phi]$ is related to an infinitesimal field redefinition

泛函 $F[\Phi]$ 与无穷小场重新定义相关

$$\Phi \mapsto \Phi + \varepsilon F[\Phi] \tag{33}$$

and can depend on the spacetime coordinates x^μ as well as on the fields Φ and their derivatives.

并且它可以依赖时空坐标 x^μ ，也可以依赖场 Φ 及其导数。

The key property of (32) is that the definition of a redundant operator requires knowledge about the equations of motion. It is instructive to illustrate this property in the context of gravity. For simplicity, we take the scaling properties to be the one of the Gaussian fixed points, i.e., all operators scale according to their classical mass dimension. Suppose Γ is given by the Einstein-Hilbert action without cosmological constant,

(32) 的核心性质是: 冗余算符的定义需要依赖运动方程的信息。我们以引力为例来说明这一性质会很有启发性。为简化起见，我们取标度性质为高斯不动点的标度性质，也就是所有算符都按照经典质量维度标度。假设 Γ 由不带宇宙学常数的爱因斯坦-希尔伯特作用量给出,

$$\Gamma[g] = -\frac{1}{16\pi G_N}\int \mathrm{d}^4x\sqrt{-g}R. \tag{34}$$

The resulting equations of motion are

由此得到的运动方程为

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 0. \tag{35}$$

Taking $F_{\mu\nu}[g] = ag_{\mu\nu}R + bR_{\mu\nu}$ in (32) then yields

将 (32) 中的 $F_{\mu\nu}[g] = ag_{\mu\nu}R + bR_{\mu\nu}$ 代入后可得

$$\mathcal{O} = \int \mathrm{d}^4x\sqrt{-g}\left[bR_{\mu\nu}R^{\mu\nu} - \frac{1}{2}(2a+b)R^2\right]. \tag{36}$$

Hence we find that two of the three possible quadratic curvature terms are redundant. In combination with the property that the third curvature-squared term can be written in terms of the topological Gauss-Bonnet integrand, this underlies the well-known result that the perturbative quantization of the Einstein-Hilbert action (34) does not require a counterterm at the one-loop level [44]. At sixth order in derivatives, the same procedure allows to eliminate all invariants except the one cubic in the Weyl tensor. It is exactly the invariant that appears at two loops in perturbation theory [45,46].

因此我们发现，三种可能的二次曲率项中有两种是冗余的。结合第三种曲率平方项可以写为拓扑高斯-博内积分的性质，这支撑了一个众所周知的结论: 爱因斯坦-希尔伯特作用量 (34) 的微扰量子化在单圈水平不需要 counterterm[44]。在六阶导数项，相同的过程可以消去除外尔张量三次项以外的所有不变量。而这个不变量正是微扰论两圈水平出现的那个不变量 [45,46]。

It is instructive to repeat this analysis for the case where Γ is given by the action for quadratic gravity. Written in the Weyl basis, we then have

当 Γ 由二次引力的作用量给出时，我们重复这一分析也很有启发性。在外尔基下写出作用量，我们得到

$$\Gamma[g] = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}\alpha C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} - \frac{1}{6}\beta R^2\right]. \tag{37}$$

The resulting equations of motion are

由此得到的运动方程为

$$\alpha\left(\nabla^\rho\nabla^\sigma+\frac{1}{2}R^{\rho\sigma}\right)C_{\mu\rho\nu\sigma}-\frac{\beta}{6}\left(R_{\mu\nu}-\frac{1}{4}g_{\mu\nu}R-\nabla_\mu\nabla_\nu+g_{\mu\nu}\nabla^2\right)R=0.$$

(38)

To remove inessential operators at sixth order in derivatives, we start from the eight independent monomials present in $d=4$. Then we again use $F_{\mu\nu}[g]=ag_{\mu\nu}R+bR_{\mu\nu}$. Evaluating (32) gives a linear combination of several monomials, and by choosing a and b , two of them can be made redundant. In quadratic gravity, we are thus left with six essential operators at this order, compared to only one starting from an Einstein-Hilbert theory.

为了移除六阶导数处的非本质算符，我们从 $d=4$ 中存在的八个独立单项式出发。之后我们再次利用 $F_{\mu\nu}[g]=ag_{\mu\nu}R+bR_{\mu\nu}$ 。对 (32) 求值会得到多个单项式的线性组合，通过选取 a 和 b ，可以让其中两个成为冗余算符。因此在二次引力中，该阶数下我们会剩下六个本质算符，而从爱因斯坦-希尔伯特理论出发仅剩下一个。

In this light, the classification presented in this section does not reference a specific dynamics which would give us information about the degrees of freedom and equations of motion of the theory. In other words, there was no attempt to identify the redundant operators in our basis, since the definition of redundant operators hinges on this input.

从这个角度看，本节给出的分类没有参考特定动力学，而特定动力学才能给出理论自由度和运动方程的相关信息。换句话说，我们没有尝试在我们的基中识别冗余算符，因为冗余算符的定义依赖这些输入信息。

Classifying Two-to-Two Scattering Processes

二对二散射过程分类

Following up on the classification given in the previous section, our interest is in physical observables related to particle scattering. The scattering amplitudes describing the most general two-to-two scattering process with external scalars and photons in a flat spacetime are readily obtained from the effective action using standard Feynman diagram techniques. Since we are working with the dressed propagators and vertices derived from the effective action, all quantum corrections are already captured by tree-level diagrams. A process then receives contributions from Yukawa-type interactions as well as matter self-interactions related to the effective four-point vertices. The corresponding diagrams are schematically depicted in Fig. 1. In this section, we summarize the most general amplitudes related to gravity-mediated scalar scattering (section ”Gravity-Mediated Scalar Scattering”), gravity-mediated photon scattering (section ”Gravity-Mediated Photon Scattering”), and mixed amplitudes involving both external scalars and photons (section ”Mixed Amplitudes”). In the case where the scalar is taken very massive, these amplitudes can be used to describe the bending of light by a massive gravitational source. Moreover, a formalism to extract corrections to the Newtonian potential based on amplitude computations has been proposed in [47-50]. Apart from the generalization considered in section ”Going Beyond Flat Spacetime”, we will work in a flat spacetime with the Minkowski metric given by

$\eta_{\mu\nu} = \mathrm{diag}(+1,-1,-1,-1)$, such that the graviton $h_{\mu\nu}$ is defined by $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. This allows us to work with momentum-space techniques.

延续上一节给出的分类，我们关注的是与粒子散射相关的物理可观测量。描述平直时空中含外部标量场与光子的最一般二对二散射过程的散射振幅，可以利用标准费曼图技术从有效作用量轻松得到。由于我们使用的是从有效作用量导出的装扮传播子与顶点，所有量子修正已经被树图所涵盖。过程的贡献来源包括汤川型相互作用，以及与有效四顶点相关的物质自相互作用。对应示意图见图 1。本节我们总结几类最一般振幅: 引力介导的标量散射 (对应“引力介导标量散射”小节)、引力介导的光子散射 (对应“引力介导光子散射”小节)，以及同时包含外部标量与光子的混合振幅 (对应“混合振幅”小节)。当标量质量极大时，这些振幅可用于描述大质量引力源导致的光线偏折。此外，文献 [47-50] 已经提出了一套基于振幅计算提取牛顿势修正的形式体系。除“超出平直时空”小节讨论的推广外，我们始终在平直时空下工作，其闵氏度规由 $\eta_{\mu\nu} = \mathrm{diag}(+1,-1,-1,-1)$ 给出，引力子 $h_{\mu\nu}$ 按 $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ 定义。这套设定允许我们使用动量空间技术进行计算。

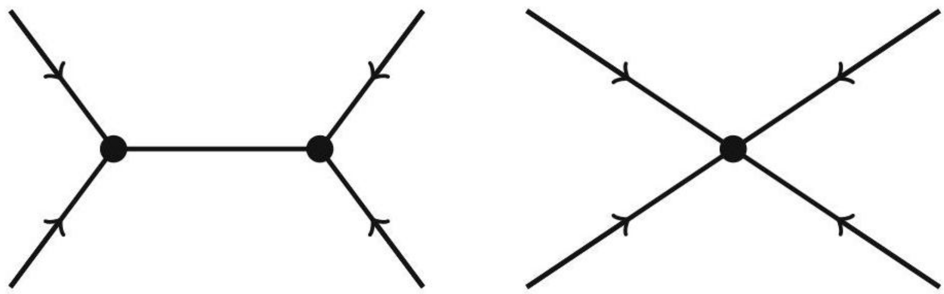

Fig. 1 Tree-level Feynman diagrams contributing to a two-to-two scattering process. They correspond to particle-mediated interactions (left) and four-point interactions (right). Each external line can either symbolize a scalar or a photon. The effective vertices are denoted by a black circle. We adopt the convention that the momentum of external particles points into the diagram

图 1 贡献二对二散射过程的树图级费曼图。它们分别对应粒子介导相互作用 (左) 和四点相互作用 (右)。每条外线可以代表标量粒子或光子。有效顶点用黑色圆圈标记。我们约定外部粒子的动量指向图内部

Kinematics of Two-to-Two Scattering Processes

二对二散射过程的运动学

As indicated by the arrows in Fig. 1, we adopt the convention that all external momenta flow into the diagram. We label these momenta by p_i^μ with $i = 1, 2$ referring to the "incoming" and $i = 3, 4$ to the "outgoing" particles. The on-shell condition for photons is

如图 1 箭头所示，我们采用所有外动量均流入费曼图的约定。我们用 p_i^μ 标记这些动量，其中 $i = 1, 2$ 对应“入射”粒子，$i = 3, 4$ 对应“出射”粒子。光子的在壳条件为

$$p^2 = 0\,. \tag{39}$$

Scalars are taken to be massive,

标量粒子假定为有质量的,

$$p^2 = m^2. \tag{40}$$

It is convenient to parameterize the amplitude in terms of the Mandelstam variables

用曼德尔斯坦变量参数化散射振幅十分方便

$$s = (p_1 + p_2)^2,\ t = (p_1 + p_3)^2,\ u = (p_1 + p_4)^2, \tag{41}$$

subject to

满足约束

$$s + t + u = p_1^2 + p_2^2 + p_3^2 + p_4^2. \tag{42}$$

Here s gives the square of the center-of-mass energy (invariant mass) and t is the square of the four-momentum transfer.

此处 s 为质心系能量的平方 (不变质量), t 是四动量转移的平方。

Denoting the spatial momenta by bold quantities, the external momenta can be parameterized by

以粗体表示空间动量,外动量可参数化为

$$p_{1\mu} = \left(\sqrt{m^2 + \mathbf{p}^2}, \mathbf{p}\right),\ p_{2\mu} = \left(\sqrt{m^2 + \mathbf{p}^2}, -\mathbf{p}\right), \tag{43}$$

$$p_{3\mu} = \left(-\sqrt{m^2 + \mathbf{q}^2}, \mathbf{q}\right),\ p_{4\mu} = \left(-\sqrt{m^2 + \mathbf{q}^2}, -\mathbf{q}\right),$$

with photons coming with $m^2 = 0$. Defining the scattering angle θ by

其中光子满足 $m^2 = 0$ 。将散射角 θ 定义为

$$\mathbf{p} \cdot \mathbf{q} = \sqrt{\mathbf{p}^2\mathbf{q}^2} \cos\theta \tag{44}$$

allows to express the Mandelstam variables t and u in terms of the center-of-mass energy s and the scattering angle. For instance, for ingoing scalars with mass m_1 and outgoing scalars with mass m_2 , one has

即可将曼德尔斯坦变量 t 和 u 表示为质心系能量 s 和散射角的函数。例如,对于质量为 m_1 的入射标量粒子和质量为 m_2 的出射标量粒子,有

$$t = -\left(\frac{s}{2} - m_1^2 - m_2^2 + \frac{1}{2}\sqrt{\left(s-4m_1^2\right)\left(s-4m_2^2\right)}\cos\theta\right), \tag{45}$$

$$u = -\left(\frac{s}{2} - m_1^2 - m_2^2 - \frac{1}{2}\sqrt{\left(s-4m_1^2\right)\left(s-4m_2^2\right)}\cos\theta\right).$$

Finally, we introduce polarization vectors for photons. The ingoing polarization vectors are taken in the $(y-z)$ -plane,

最后，我们引入光子的极化矢量。入射极化矢量取在 $(y-z)$ 平面内,

$$e_\mu^{\text{in}+} = \frac{1}{\sqrt{2}}(0,0,1,-i), \tag{46}$$

and the outgoing polarization vectors are

出射极化矢量为

$$e_\mu^{\text{out}+} = \frac{1}{\sqrt{2}}(0,-\sin\theta,\cos\theta,-i). \tag{47}$$

The parameterizations in Eqs. (43), (46), and (47) allow to work out all scalar products between momentum four vectors and polarization tensors in terms of either the scattering angle and spatial momenta or, equivalently, using the Mandelstam variables [28].

式 (43)、(46) 和 (47) 的参数化方法可以将所有四动量与极化张量之间的标积全部推导为散射角和空间动量的函数，或者等价地用曼德尔斯坦变量表示 [28]。

Amplitudes from the Quantum Effective Action

量子有效作用量的振幅

In this section, we will list the scattering amplitudes of two-to-two particle processes parameterized by the effective action presented in section "The Quantum Effective Action Including Form Factors". We will begin with scalar scattering in section "Gravity-Mediated Scalar Scattering". We then continue with photon scattering in section "Gravity-Mediated Photon Scattering" and conclude our exposition in section "Mixed Amplitudes" with scalar-photon scattering.

在本节中，我们将列出由包含形状因子的量子有效作用量一节中给出的有效作用量参数化的二对二粒子过程的散射振幅。我们将在引力介导标量散射一节中从标量散射开始讨论，随后在引力介导光子散射一节中分析光子散射，最后在混合振幅一节中介绍标量-光子散射。

Gravity-Mediated Scalar Scattering

引力介导的标量散射

We begin our list of scattering processes with scalar scattering. The total amplitude for the process $\phi\phi \to \phi\phi$ is given by [26]

我们从标量散射开始介绍散射过程列表。过程 $\phi\phi \to \phi\phi$ 的总振幅由文献 [26] 给出

$$\mathcal{A}^{\phi} = \mathcal{A}_s^{\phi} + \mathcal{A}_t^{\phi} + \mathcal{A}_u^{\phi} + \mathcal{A}_4^{\phi}. \tag{48}$$

This corresponds to the scattering amplitude of the s, t , and u channels, as well as the four-point diagram.

这对应 s, t 道、u 道以及四点图的散射振幅。

The s -channel amplitude for scalar-to-scalar scattering reads

标量到标量散射的 s 道振幅为

$$\mathcal{A}_s^{\phi} = \frac{4\pi G_N}{3}\Big[\big((s+2m^2)\left(1+sf_{\mathrm{Ric}\phi\phi}\right) - 12sf_{R\phi\phi}\big)^2 G_{RR}(s)$$

(49)

$$-\left(1+sf_{\mathrm{Ric}\,\phi\phi}\right)^2 G_{CC}(s)\left\{t^2-4tu+u^2\right\}\Big]\,.$$

Here we have evaluated the form factors $f_{\mathrm{Ric}\phi\phi}$ and $f_{R\phi\phi}$ at

此处我们计算了形状因子 $f_{\mathrm{Ric}\phi\phi}$ 和 $f_{R\phi\phi}$ 在

$$f_{\mathrm{Ric}\phi\phi} = f_{\mathrm{Ric}\phi\phi}\left(s, m^2, m^2\right) \quad \text{and} \quad f_{R\phi\phi} = f_{R\phi\phi}\left(s, m^2, m^2\right). \tag{50}$$

Furthermore, we have introduced the functions

此外，我们定义了函数

$$G_{RR}(x) = \frac{1}{x\left(1+xf_{RR}(x)\right)},\; G_{CC}(x) = \frac{1}{x\left(1+xf_{CC}(x)\right)}, \tag{51}$$

related to the graviton propagator. We obtain the t and u channels from (49) by crossing symmetry, interchanging $s \leftrightarrow t$ and $s \leftrightarrow u$, respectively.

其与引力子传播子相关。我们通过交叉对称性，分别交换 $s \leftrightarrow t$ 和 $s \leftrightarrow u$ ，从式 (49) 得到了 t 道和 u 道。

The four-point diagram is given by

四点图的贡献为

$$\mathcal{A}_4^{\phi} = f_{\phi^4}\left(\frac{s-2m^2}{2}, \frac{t-2m^2}{2}, \frac{u-2m^2}{2}, \frac{u-2m^2}{2}, \frac{t-2m^2}{2}, \frac{s-2m^2}{2}\right).$$

(52)

This concludes the description of the scalar scattering amplitude.

以上就是对标量散射振幅的完整描述。

Gravity-Mediated Photon Scattering

引力介导光子散射

We will now consider four-photon scattering. Again, we have particle-mediated and four-point contributions to the scattering amplitude. For the particle-mediated diagram, the exchanged particle is either a photon or a graviton. Computing the vertices arising from the action (26) and setting the two external photon legs on-shell show that the three-photon vertices vanish. Therefore, the only contribution comes from a graviton-exchanged diagram.

我们现在研究四光子散射。和之前一样，散射振幅存在粒子传播项和四点贡献项。对于粒子传播图，交换粒子要么是光子，要么是引力子。计算从作用量 (26) 得到的顶点，并将两个外光子腿置于在壳条件后，可以发现三光子顶点为零。因此，唯一的贡献来自引力子交换图。

The full amplitude for this process is given by [28]

该过程的完整振幅由文献 [28] 给出

$$\mathcal{A}^{\gamma} = \mathcal{A}_s^{\gamma} + \mathcal{A}_t^{\gamma} + \mathcal{A}_u^{\gamma} + \mathcal{A}_4^{\gamma}. \tag{53}$$

The amplitudes can be organized by their helicity configurations. We have the following classes:

振幅可以按螺旋度构型分类，我们得到以下几类:

$$\begin{gathered}
\text{I:}\ \mathcal{A}^{+--+} = \mathcal{A}^{-++-}\,, \\
\text{II:}\ \mathcal{A}^{++++} = \mathcal{A}^{----}\,, \\
\text{III:}\ \mathcal{A}^{++--} = \mathcal{A}^{--++}\,, \\
\text{IV:}\ \mathcal{A}^{+-+-} = \mathcal{A}^{-+-+}\,, \\
\text{V:}\ \mathcal{A}^{+++-} = \mathcal{A}^{++-+} = \mathcal{A}^{+-++} = \mathcal{A}^{-+++} = \\
\mathcal{A}^{---+} = \mathcal{A}^{--+-} = \mathcal{A}^{-+--} = \mathcal{A}^{+---}.
\end{gathered} \tag{54}$$

The t -channel contributions to these expressions read

这些表达式的 t 道贡献如下

$$\mathrm{I}: \mathcal{A}_t^{\gamma} = 2\pi G_N s^2 \left(-2 + t^2 f_{D^2\,\mathrm{Ric}\,FF} - 2t f_{\mathrm{Ric}\,FF} - 4t f_{\mathrm{Rm}\,FF}\right)^2$$

$$G_{CC}(t)\,, \tag{55}$$

$$\mathrm{II} = \mathrm{IV}:\ \mathcal{A}_t^{\gamma} = \frac{\pi}{3} G_N t^2 \left(-t f_{D^2\,\mathrm{Ric}\,FF} + 4 f_{\mathrm{Rm}\,FF} - f'_{FF}(0)\right)^2$$

$$\times \left(s^2 - 4su + u^2\right) G_{CC}(t)$$

$$-\frac{\pi}{3} G_N t^4 \left(6t f_{D^2 RFF} + t f_{D^2\,\mathrm{Ric}\,FF} - 24 f_{RFF}\right.$$

$$\left.-6 f_{\mathrm{Ric}FF} - 4 f_{\mathrm{Rm}FF} - 2 f'_{FF}(0)\right)^2 G_{RR}(t)\,,$$

(56)

$$\mathrm{III}: \mathcal{A}_t^{\gamma} = 2\pi G_N u^2 \left(-2 + t^2 f_{D^2\,\mathrm{Ric}\,FF} - 2t f_{\mathrm{Ric}\,FF} - 4t f_{\mathrm{Rm}\,FF}\right)^2 G_{CC}(t)\ ,$$

(57)

$$\mathrm{V}: \mathcal{A}_t^{\gamma} = 2\pi G_N su \left(-t^2 f_{D^2\,\mathrm{Ric}\,FF} + 4t f_{\mathrm{Rm}\,FF} - t f'_{FF}(0)\right)$$

$$\times \left(-2 + t^2 f_{D^2\,\mathrm{Ric}\,FF} - 2t f_{\mathrm{Ric}\,FF} - 4t f_{\mathrm{Rm}\,FF}\right) G_{CC}(t)\,.$$

(58)

Here the arguments of the form factors are suppressed in the following way:

这里我们按如下方式省略形状因子的自变量:

$$f_{RFF} = f_{RFF}(t,0,0)\,,\ f_{\mathrm{Ric}\,FF} = f_{\mathrm{Ric}\,FF}(t,0,0)\,,$$

$$f_{D^2 RFF} = f_{D^2 RFF}(t,0,0)\,,\ f_{D^2\,\mathrm{Ric}\,FF} = f_{D^2\,\mathrm{Ric}\,FF}(t,0,0)\,,$$

$$f_{\mathrm{Rm}FF} = f_{\mathrm{Rm}FF}(0,0)\,.$$

(59)

We now present the four-point amplitudes. Since these expressions are rather lengthy, we summarize them in Table 1.

我们现在给出四点振幅。由于这些表达式相当冗长，我们将其汇总在表 1 中。

Table 1 Different contributions of the four-photon vertex to $\mathcal{A}_d^{\gamma}$. The function f should be read as $f(a,b,c) = f_I\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}, \frac{c}{2}, \frac{b}{2}, \frac{a}{2}\right)$. By [perm.], we denote the sum of $f(s,t,u)$ and the five permutations of its arguments. The 16 polarization configurations can be obtained by interchanging $(+ \leftrightarrow -)$ following the scheme in (54). The total amplitude for each polarization configuration is obtained by summing the contribution from each form factor in the respective column. (From [28])

表 1 四光子顶点对 $\mathcal{A}_d^\gamma$ 的不同贡献。函数 f 应理解为 $f(a,b,c) = f_I\left(\frac{a}{2},\frac{b}{2},\frac{c}{2},\frac{c}{2},\frac{b}{2},\frac{a}{2}\right)$。我们用 [perm.] 表示 $f(s,t,u)$ 及其自变量的五种排列的总和。16 种极化构型可以按照 (54) 中的方案交换 $(+ \leftrightarrow -)$ 得到。每种极化构型的总振幅可通过对对应列中每个形状因子的贡献求和得到。(引自 [28])

	I $+--+$	II +-+++	III $++--$	IV +-+-	V ++-+-
$f_{F^2F^2}$	$4s^2\left[f(s,t,u)+f(s,u,t)\right]$	$4t^2\left[f(t,s,u)+f(t,u,s)\right]$	$4u^2\left[f(u,s,t)+f(u,t,s)\right]$	I + II + III $-2\{su\left[f(s,t,u)+f(u,t,s)\right]]$	0
f_{F^4}	s^2 [perm.]	t^2 [perm.]	u^2 [perm.]	$+st\left[f(s,u,t)+f(t,u,s)\right]$ $+tu\left[f(t,s,u)+f(u,s,t)\right]\}$	0
f_{FFDFDF_1}	$\frac{1}{2}s^2\{t\left[f(s,t,u)-f(t,s,u)\right]$ $+u\left[f(s,u,t)-f(u,s,t)\right]\}$	$\frac{1}{2}t^2\{s\left[f(t,s,u)-f(s,t,u)\right]$ $+u\left[f(t,u,s)-f(u,t,s)\right]\}$	$\frac{1}{2}u^2\{s\left[f(u,s,t)-f(s,u,t)\right]$ $+t\left[f(u,t,s)-f(t,u,s)\right]\}$	$\frac{1}{2}$ stu[perm.]	0
f_{FFDFDF_2}	0	0	0	0	0
f_{FFDFDF_3}	0	0	0	stu[perm.]	$\frac{1}{4}$ stu[perm.]
f_{FFDFDF_4}	$s^2\left[tf(s,u,t)+uf(s,t,u)\right]$	$t^2\left[sf(t,s,u)+uf(t,u,s)\right]$	$u^2\left[sf(u,t,s)+tf(u,s,t)\right]$	I + II + III	$-\frac{1}{2}$ stu[perm.]
$f_{FFD^2FD^2F}$	$-\frac{1}{2}s^2tu\left[f(s,t,u)+f(s,u,t)\right]$	$-\frac{1}{2}t^2su\left[f(t,s,u)+f(t,u,s)\right]$	$-\frac{1}{2}u^2st\left[f(u,s,t)+f(u,t,s)\right]$	I + II + III	$\frac{1}{2}\cdot$ IV

Mixed Amplitudes

混合振幅

Finally, we consider the process $\gamma\phi \to \gamma\phi$. Since the two particles in the initial and final states are distinguishable, only the t channel will give a contribution to the exchange diagrams. The total amplitude is thus given by

最后，我们考虑过程 $\gamma\phi \to \gamma\phi$ 。由于初态和末态的两个粒子可区分，仅 t 道会对交换图产生贡献。因此总振幅由下式给出

$$\mathcal{A}^{\gamma\phi} = \mathcal{A}_t^{\gamma\phi} + \mathcal{A}_4^{\gamma\phi}. \tag{60}$$

The t -channel amplitude is given by

t 道振幅由下式给出

$$\mathcal{A}_t^{++} = -\frac{2\pi G_N}{3}\left(1 - tf_{\mathrm{Ric}D\phi D\phi}\right)\left(tf_{D^2\mathrm{Ric}FF} - 4f_{\mathrm{Rm}FF} + f'_{FF}(0)\right)$$

$$\times\left(s^2 - 4su + u^2 + 2m^4\right)tG_{CC}(t)$$

$$+\frac{2\pi G_N}{3}t^2\left(t + 2m^2 + 2t\left(t - m^2\right)f_{\mathrm{Ric}D\phi D\phi} - 12tf_{R\phi\phi}\right)G_{RR}(t)$$

$$\times\left(6tf_{D^2RFF} + tf_{D^2\,\mathrm{Ric}\,FF} - 24f_{RFF} - 6f_{\mathrm{Ric}\,FF} - 4f_{\mathrm{Rm}\,FF} - 2f'_{FF}(0)\right),$$

(61)

$$\mathcal{A}_t^{+-} = 4\pi G_N \left(1 - t f_{\mathrm{Ric}D\phi D\phi}\right)\left(-2 + t^2 f_{D^2\mathrm{Ric}FF} - 2t f_{\mathrm{Ric}FF} - 4t f_{\mathrm{Rm}FF}\right)$$

$$\times\left(su - m^4\right) G_{CC}(t) \ .$$

(62)

Here we have suppressed the arguments of the form factors as in (59), and in addition

此处我们同式 (59) 一样省略了形状因子的自变量，此外

$$f_{R\phi\phi} = f_{R\phi\phi}\left(t, m^2, m^2\right), \ f_{\ \mathrm{RicD}\ \phi D\phi} = f_{\ \mathrm{RicD}\ \phi D\phi}\left(t, m^2, m^2\right). \tag{63}$$

We continue with the four-point amplitude. This is given by

接下来我们讨论四点振幅，其表达式为

$$\mathcal{A}_4^{++} = \frac{1}{4} t \left[8 f_{FF\phi\phi}(t,s,u) - 2s f_{FFD\phi D\phi}(t,s,u) + 2m^2 f_{FFD^2\phi\phi}(t,s,u)\right.$$

$$\left. - \left(su - m^4\right) f_{FFD^2\phi D^2\phi}(t,s,u)\right] + (s \leftrightarrow u),$$

(64)

$$\mathcal{A}_4^{+-} = \frac{1}{4}\left(su - m^4\right)\left[2 f_{FFD\phi D\phi}(t,s,u) - 2 f_{FFD^2\phi\phi}(t,s,u)\right. \tag{65}$$

$$\left. - t f_{FFD^2\phi D^2\phi}(t,s,u)\right] + (s \leftrightarrow u).$$

(66)

Here, we suppressed half of the arguments of the form factors following the prescription

此处我们遵循约定省略了形状因子一半的自变量

$$f_{PQRS}(a,b,c) = f_{PQRS}\left(\frac{a}{2}, \frac{b-m^2}{2}, \frac{c-m^2}{2}, \frac{c-m^2}{2}, \frac{b-m^2}{2}, \frac{a}{2} - m^2\right).$$

(67)

From the amplitudes $\mathcal{A}^{++}$ and $\mathcal{A}^{+-}$, the two remaining helicity configurations can then be obtained by parity transformations. This gives

由振幅 $\mathcal{A}^{++}$ 和 $\mathcal{A}^{+-}$ ，剩余两个螺旋度构型可通过宇称变换得到，结果如下

$$\lambda^{++} = \lambda^{--}, \ \lambda^{+-} = \lambda^{-+}. \tag{68}$$

Finally, the scattering amplitude of the process $\phi\phi \to \gamma\gamma$ can be obtained by using crossing symmetry. Since the polarization channels do not change, we obtain the scattering amplitude from (60) by exchanging $t \leftrightarrow s$. This concludes our listing of scattering amplitudes.

最后，过程 $\phi\phi \to \gamma\gamma$ 的散射振幅可通过交叉对称性得到。由于极化道不发生改变，我们通过交换 $t \leftrightarrow s$ 即可从式 (60) 得到该散射振幅。至此我们完成了所有散射振幅的列举。

Going Beyond Flat Spacetime

超越平直时空

A natural generalization of the amplitude formalism developed above is to go beyond a flat spacetime. Promoting scattering amplitudes to a curved spacetime allows us to study aspects of the nonlinear coupling of quantum fields to gravity. In section ”Challenges in QFT on a Curved Spacetime”, we discuss some of the challenges that arise in the formulation of QFT on a curved spacetime. Many of these problems are already present in the simplest setting where spacetime exhibits a constant scalar curvature. We present some recent developments in the description of scattering amplitudes in such a spacetime in section ”Scattering Amplitudes in de Sitter Spacetime”.

将上文建立的散射振幅形式化方法自然推广，即可突破平直时空的限制。把散射振幅推广到弯曲时空，能让我们研究量子场与引力非线性耦合的相关内容。在“弯曲时空量子场论的挑战”小节中，我们讨论了在弯曲时空构建量子场论时出现的若干挑战。其中很多问题已经出现在时空具有常数标量曲率的最简单场景中。我们在“德西特时空的散射振幅”小节中介绍了此类时空下散射振幅描述的最新进展。

Challenges in QFT on a Curved Spacetime

弯曲时空下量子场论的挑战

The generalization of QFT from Minkowski spacetime to curved spacetime comes with a number of major challenges. Here we highlight two: the absence of noninteracting asymptotically flat boundaries and the lack of spacetime symmetries.

将量子场论从闵可夫斯基时空推广到弯曲时空会带来诸多重大挑战。在此我们着重介绍两点: 不存在无相互作用的渐近平直边界，以及缺乏时空对称性。

Generic curved spacetimes do not possess an asymptotically flat boundary. Solutions to the wave equation interact at any time with the gravitational field and are not propagating freely asymptotically: It can be shown that even one-particle states can decay [51-54]. As a consequence, it is problematic to describe a scattering process as well-defined particles entering the scattering region and interacting for a limited time. This is also the main obstruction to constructing an S -matrix [55-59].

一般的弯曲时空不具备渐近平直边界。波动方程的解在任意时刻都与引力场相互作用，不会在渐近区域自由传播: 可以证明，即便是单粒子态也会发生衰变 [51-54]。因此，我们很难将散射过程描述为定义明确的粒子进入散射区域并在有限时间内发生相互作用。这也是构建 S 矩阵的主要障碍 [55-59]。

Another complication in the formulation of QFT in a curved spacetime is related to a potential lack of symmetries. In Minkowski spacetime, there is a unique distinguished vacuum state that is Poincaré invariant. Particles defined with respect to this vacuum are characterized by conserved energy momentum charges. In general, such a state does not exist in a curved setting, and particles cannot be classified according to conserved quantities. Also, at a computational level, amplitudes in Minkowski spacetime are most conveniently computed using momentum-space techniques. Due to the noncommutativity of covariant derivatives, there exists no momentum space in curved spacetime. This is another major obstacle in the concrete calculation of curved-spacetime scattering amplitudes.

在弯曲时空建立量子场论的另一难点源于对称性的缺失。在闵可夫斯基时空，存在一个唯一的、特殊的庞加莱不变真空态。相对于该真空定义的粒子由守恒的能动量荷表征。一般而言，这样的真空态在弯曲时空背景中并不存在，粒子也无法通过守恒量分类。同时，在计算层面，闵可夫斯基时空的振幅最适合用动量空间技术计算。由于协变导数不对易，弯曲时空不存在动量空间。这是具体计算弯曲时空散射振幅的另一大障碍。

Scattering Amplitudes in de Sitter Spacetime

德西特时空下的散射振幅

De Sitter spacetime serves as a benchmark for the construction of scattering amplitudes in curved spacetime. This maximally symmetric space is characterized by constant positive scalar curvature $R = 12H^2$, where H is Hubble’ s constant, while all other curvature tensors vanish.

德西特时空是构造弯曲时空散射振幅的基准场景。这个最大对称空间的特征是具有恒定正标量曲率 $R = 12H^2$ ，其中 H 是哈勃常数，所有其他曲率张量均为零。

QFT in de Sitter spacetime already exhibits many of the problems present in the construction of QFT in a general curved spacetime, and in particular the challenges posed above. However, due to the relatively simple curvature structure, properties such as the (non-)uniqueness of the vacuum can be studied explicitly.

德西特时空中的量子场论已经展现出一般弯曲时空量子场论构造中的诸多问题，尤其是前文提到的各类挑战。但由于曲率结构相对简单，真空的 (非) 唯一性这类性质可以被显式研究。

The simple curvature structure also lends itself for the form factor framework. Since any derivative of the spacetime curvature vanishes, we can classify interactions in an expansion about a constantly curved reference point instead of about Minkowski spacetime. The form factors are now characterized by an additional curvature parameter. For example, the $R\phi\phi$ -form factor generalizes to

简洁的曲率结构也适配形状因子框架。由于时空曲率的任意导数都为零，我们可以围绕恒定曲率参考点而非闵氏时空做展开来对相互作用分类。形状因子现在由一个额外的曲率参数表征。例如，$R\phi\phi$ 形状因子被推广为

$$\int \mathrm{d}^4 x \sqrt{-g} f^{\mathrm{dS}}_{R\phi\phi}(R, \Box_1, \Box_2, \Box_3) R\phi\phi. \tag{69}$$

This action contains the interactions that contribute to the flat three-point vertex, since setting $f_{R\phi\phi}(\Box_1,\Box_2,\Box_3) = f^{\mathrm{dS}}_{R\phi\phi}(0,\Box_1,\Box_2,\Box_3)$ gives the $R\phi\phi$ -term contained in $\Gamma_{h\phi^2}$ in (18). One has to keep in mind, however, that the form factor in (69) overlaps with the generalized kinetic term. The extraction of vertices and propagators in de Sitter spacetime is therefore a subtle question.

该作用量包含了对平直三点顶点有贡献的相互作用，因为令 $f_{R\phi\phi}(\Box_1,\Box_2,\Box_3) = f^{\mathrm{dS}}_{R\phi\phi}(0,\Box_1,\Box_2,\Box_3)$ 就可得到 (18) 中 $\Gamma_{h\phi^2}$ 包含的 $R\phi\phi$ 项。但必须注意，(69) 中的形状因子与推广后的动能项存在重叠。因此，德西特时空中顶点和传播子的提取是一个微妙问题。

At this point, a remark about the cosmological constant Λ is in order. When previously working in a flat spacetime, we have explicitly set Λ to zero. This ensured that the Minkowski spacetime is a solution to the gravitational equations of motion. In order to render de Sitter spacetime an on-shell solution, the inclusion of a positive Λ is required.

在此，有必要对宇宙学常数 Λ 做一点说明。此前我们在平直时空工作时，已经显式地将 Λ 设为零，这保证了闵氏时空是引力运动方程的一个解。为了让德西特时空成为一个在壳解，需要引入正的 Λ 。

Having discussed the problems related to the classification of the interactions that contribute to a de Sitter spacetime amplitude, the next task would be to compute the associated tree-level Feynman diagrams. Conceptually, this is a straightforward generalization of the procedure in flat spacetime: Vertices and propagators are obtained from Γ by taking functional derivatives with respect to the fields, while the Feynman diagram is constructed from the contraction of the vertices and propagators.

在讨论完与德西特时空振幅相关的相互作用分类问题后，下一个任务是计算对应的树级费曼图。从概念上看，这是平直时空流程的直接推广: 顶点和传播子可以通过对场求泛函导数从 Γ 得到，再通过收缩顶点和传播子构造费曼图。

In order to bring the scattering amplitude in a manifestly gauge-invariant and on-shell form, we have to carefully take into account the commutator of covariant derivatives with form factors. Techniques for handling such commutators were developed in the context of affine gravity in [60] and adapted to metric gravity in [61].

为了让散射振幅具有明显的规范不变性和在壳形式, 我们必须仔细考虑协变导数与形状因子的对易子。处理这类对易子的技术最早在仿射引力的背景下于文献 [60] 中提出，后在文献 [61] 中被适配到度规引力。

Using these techniques, we can compute a scattering amplitude in terms of differential operators. In the case of a conformally coupled massless scalar field in Einstein-Hilbert gravity, the tree-level scattering amplitude functional of the graviton-mediated process $\phi\phi \to \phi\phi$ in de Sitter spacetime reads

利用这些技术，我们可以用微分算子表示计算散射振幅。对于爱因斯坦-希尔伯特引力中共形耦合的无质量标量场，德西特时空中引力子介导过程 $\phi\phi \to \phi\phi$ 的树级散射振幅泛函为

$$\mathcal{A}^{\phi}_{\mathrm{dS}} = 16\pi G_N \int \mathrm{d}^4x\sqrt{-g}\, T_{\alpha\beta}\left[\phi_1,\phi_2\right] \mathcal{G}(\Box)\, T^{\alpha\beta}\left[\phi_3,\phi_4\right]. \tag{70}$$

Here the propagator $\mathcal{G}(\Box)$ is given by

此处传播子 $\mathcal{G}(\Box)$ 由下式给出

$$\mathcal{G}(\Box) = \left(\Box + 2H^2\right)^{-1}, \tag{71}$$

while the on-shell vertex is captured by the tensor

而在壳顶点由如下张量描述

$$T_{\mu\upsilon}[\phi_1, \phi_2] = \left(\nabla_{(\mu}\phi_1\right)\left(\nabla_{\upsilon)}\phi_2\right) - \frac{1}{4}g_{\mu\upsilon}\left(\nabla_\gamma\phi_1\right)\left(\nabla^\gamma\phi_2\right)$$

$$-\frac{1}{6}\left[\nabla_\mu\nabla_\nu + \frac{1}{4}g_{\mu\nu}\Box\right]\phi_1\phi_2. \tag{72}$$

Comparing this expression to the scattering amplitude in flat spacetime, we see that covariant derivatives play the role of generalized momenta. We notice, however, that these are fully non-commuting operators due to the finite spacetime curvature.

将该表达式与平直时空的散射振幅对比，我们可以发现协变导数扮演了推广动量的角色。但我们注意到，由于存在有限的时空曲率，这些算子是完全非对易的。

Due to the differential operator nature of (71), computing the number given by (70) is less straightforward than in flat spacetime. In [61], the scattering amplitude of massive scalars in Einstein-Hilbert gravity was computed in the adiabatic limit, where the scalar mass m is much larger than the Hubble constant H . This is the de Sitter spacetime analog of taking the nonrelativistic limit. By Fourier-transforming the resulting amplitude, one obtains the Newtonian potential V . For small separations r , this reproduces the standard $-1/r$ potential, in accordance with flat spacetime. For separations $r \lesssim H$, one obtains curvature corrections that can be interpreted as a repulsive force. This complies with the picture of de Sitter spacetime as an exponentially expanding FLRW universe. For super-Hubble radii $r > H$, the potential is identically zero, thereby making manifest that fields separated by the de Sitter horizon are not in causal contact.

由于 (71) 具有微分算子性质，计算 (70) 给出的数值比平直时空下更复杂。在文献 [61] 中，爱因斯坦-希尔伯特引力下有质量标量的散射振幅是在绝热极限下计算的，该极限中标量质量 m 远大于哈勃常数 H 。这对应非相对论极限在德西特时空中的类比。对得到的振幅做傅里叶变换，即可得到牛顿势 V 。对于小间距 r ，该结果会重现标准的 $-1/r$ 势，与平直时空的结果一致。对于间距 $r \lesssim H$ ，可以得到曲率修正，该修正可以被解释为一种排斥力，这与德西特时空是指数膨胀 FLRW 宇宙的图像一致。对于超哈勃半径 $r > H$ ，势恒为零，这清楚地表明被德西特视界分隔的场不存在因果联系。

Asymptotically Safe Scattering Amplitudes

渐近安全散射振幅

The amplitudes constructed in section ”Classifying Two-to-Two Scattering Processes” constitute the most general result for scattering in a flat spacetime compatible with a relativistic QFT. The goal of this section is to explore Asymptotic Safety within this general framework. Our discussion focuses on the gravity-mediated scattering of scalar particles in a flat spacetime, building on the results of section ”Gravity-Mediated Scalar Scattering”. We start by summarizing the properties of asymptotically safe amplitudes in section ”Definition of an Asymptotically Safe Amplitude” before providing instructive examples in section ”Realizing Asymptotically Safe Amplitudes via Form Factors”. Some general remarks are added in section ”Anomalous Dimensions Within the Form Factor Framework”.

在“二对二散射过程分类”一节中构建的振幅，是相对论量子场论相容的平直时空散射最具一般性的结果。本节的目标是在该通用框架内探究渐近安全。我们基于“引力介导标量散射”一节的结果，重点讨论平直时空中由引力介导的标量粒子散射。我们首先在“渐近安全振幅的定义”一节总结渐近安全振幅的性质，随后在“通过形状因子实现渐近安全振幅”一节给出具有启发性的示例，最后在“形状因子框架内的反常维数”一节补充若干一般性讨论。

Our exposition is limited to scattering processes with four external fields. This implies that phenomena related to IR divergences in the amplitudes, whose cure is expected to result from the resummation of diagrams including an arbitrary number of soft external gravitons [62], are beyond the scope of the exposition. Similarly, discussing the back reaction of the scattered particles onto the geometry is also beyond the scope of this chapter, as incorporating this effect in a flat spacetime may also require adding additional gravitons. While this renders the exposition incomplete in some aspects, it nevertheless highlights the importance of form factors for the gravitational Asymptotic Safety program.

我们的阐述仅限于包含四个外场的散射过程。这意味着，与散射振幅中的红外发散相关的现象不在本文讨论范围内，这类发散预期可通过对包含任意数量软外引力子的图进行重求和得到解决 [62]。类似地，讨论散射粒子对几何的反作用也超出了本章范围，因为在平直时空引入该效应也可能需要添加额外引力子。尽管这使得我们的阐述在某些方面不够完整，但它仍凸显了形状因子对引力渐近安全项目的重要性。

Definition of an Asymptotically Safe Amplitude

渐近安全散射振幅的定义

The idea that gravity could be asymptotically safe appeared in the seminal work by Weinberg [4]. This essay provided the following characterization of the Asymptotic Safety mechanism: ”A theory is said to be asymptotically safe if the essential coupling parameters approach a fixed point as the momentum scale of their renormalization point goes to infinity. This condition is introduced [...] as a means of avoiding unphysical singularities at very high energy.” This statement of intent can actually be found in many works exploring the viability of this scenario by studying the Wilsonian RG flow for gravity and gravity-matter systems.

引力可以具有渐近安全性的观点最早出现在温伯格的开创性工作 [4] 中。该文章对渐近安全机制给出了如下描述:「当重整化能标的动量标度趋于无穷时，基本耦合参数趋近于一个不动点，这样的理论就是渐近安全的。引入这一条件是为了避免极高能下出现非物理奇点。」在许多通过研究引力及引力-物质系统的威尔逊重整化群流，探究该场景可行性的工作中，都能找到这一核心表述。

The scattering amplitudes constructed in the previous section constitute prototypical entities which should be free from unphysical divergences. Formulating Asymptotic Safety in terms of the form factor framework then allows us to make this statement of intent precise in an operational way. The notion of essential couplings translates to the combination of form factors that appear within an on-shell amplitude. The fact that the amplitudes are gauge-invariant illustrates that not all combinations of form factors are essential in this sense: Since the gauge fixing does not appear in the amplitude, it is inessential according to this definition. The concept of the momentum scale of an essential coupling is captured by the generalized momentum dependence of the form factors. Notably, the form factor framework captures dynamics going beyond a "running" coupling depending on a single momentum scale. In particular, it encodes the general momentum dependence of the n -point vertices on all independent combinations of the external momenta.

前一节构造的散射振幅是应当不存在非物理发散的典型实体。将渐近安全用形状因子框架表述，能让我们在可操作层面明确这一核心表述。基本耦合的概念对应到壳散射振幅中出现的形状因子组合。并非所有形状因子组合在此意义下都是基本的，这一点可以从振幅的规范不变性看出: 由于规范固定项不出现在振幅中，按照定义它就是非基本的。基本耦合动量标度的概念由形状因子的广义动量依赖体现。值得注意的是，形状因子框架可以描述依赖单个动量标度的「跑动」耦合之外的动力学，具体来说，它可以编码 n 点顶点对所有独立外动量组合的一般动量依赖。

Asymptotic Safety manifests itself as constraints on the momentum dependence of the combinations building essential form factors: They should remain bounded as the center-of-mass energy goes to infinity. This constraint applies to all amplitudes that can be constructed from a given field content. This implies, in particular, that all amplitudes are bounded in the Jin-Martin sense [63], satisfying

渐近安全体现为对构成基本形状因子的组合的动量依赖施加约束: 当质心系能量趋于无穷时，这些组合应当保持有界。该约束适用于所有可由给定场内容构造出的振幅，这意味着所有振幅都满足金-马丁意义下的有界性 [63]，即

$$\lim_{|s|\to\infty} s^{-2}\mathcal{A}(s,t) = 0. \tag{73}$$

The necessary cancellations require a delicate interplay between the form factors making up the propagators and vertices. It is this point where the RG fixed points underlying Asymptotic Safety come into play. If the high-energy behavior of a theory is controlled by such a UV fixed point, quantum scale invariance may precisely provide the necessary relations for canceling the unphysical divergences.

所需的抵消要求传播子和顶点对应的形状因子之间存在精细的相互作用，渐近安全背后的重整化群不动点正是在这一点发挥作用。如果理论的高能行为由这样一个紫外不动点控制，量子标度不变性恰好可以给出抵消非物理发散所需的关系。

Realizing Asymptotically Safe Amplitudes via Form Factors

利用形状因子实现渐近安全散射振幅

For concreteness, let us consider the gravity-mediated scattering of two massless scalar particles ϕ, χ with a standard kinetic term,

为具体起见，我们考虑具有标准动能项的引力介导的两个无质量标量粒子散射 ϕ, χ ,

$$f_{\phi\phi}(\Box) = f_{\chi\chi}(\Box) = \Box. \tag{74}$$

Throughout the discussion in this subsection, we assume that the scalars are minimally coupled to gravity, so that all form factors associated with non-minimal gravity-matter vertices vanish,

在本小节的整个讨论中，我们假设标量与引力最小耦合，因此所有与非最小引力-物质顶点相关的形状因子都为零,

$$f_{R\phi\phi} = f_{\mathrm{Ric}\,\phi\phi} = f_{R\chi\chi} = f_{\mathrm{Ric}\,\chi\chi} = 0. \tag{75}$$

Let us first focus on the two-to-two scattering process $\phi\phi \to \chi\chi$. In this case, the scattering amplitude $\mathcal{A}_s(s,\theta)$ is encoded by a single s -channel diagram whose topology is given by the left diagram of Fig. 1:

我们首先聚焦于二对二散射过程 $\phi\phi \to \chi\chi$ 。在这种情况下，散射振幅 $\mathcal{A}_s(s,\theta)$ 由单个 s 通道图描述，其拓扑结构如图 1 左侧图所示:

$$\mathcal{A}_s(s,\theta) = \frac{4\pi G_N}{3} s^2 \left[G_0(s) - P_2(\cos\theta)\, G_2(s)\right]. \tag{76}$$

Here θ is the scattering angle in the center-of-mass frame, and $P_j(x)$ are the standard Legendre polynomials of order j with $P_2(x) \equiv \left(3x^2-1\right)/2$. The scalar parts of the spin-zero and spin-two propagators including the form factor contributions (16) are

此处 θ 是质心系中的散射角，$P_j(x)$ 是满足 $P_2(x) \equiv \left(3x^2-1\right)/2$ 的 j 阶标准勒让德多项式，包含形状因子贡献 (16) 的自旋零和自旋二传播子的标量部分为

$$G_0^{-1}\left(p^2\right) = \left(p^2 + \mathrm{i}\varepsilon\right)\left(1 + p^2 f_{RR}\left(p^2\right)\right), \tag{77}$$

$$G_2^{-1}\left(p^2\right) = \left(p^2 + \mathrm{i}\varepsilon\right)\left(1 + p^2 f_{CC}\left(p^2\right)\right).$$

The massless pole has been equipped with the standard ”iε” prescription for a Feynman propagator. The form factor contributions may have similar terms in order to give the correct prescription for poles and branch cuts. These terms are left implicit.

无质量极点已经按费曼传播子采用了标准的”iε” 约定。形状因子贡献也可以包含类似项，为极点和分支切割给出正确约定，这些项在此隐去不写。

The s -channel scattering process is conveniently analyzed using the partial wave decomposition

s 通道散射过程可以方便地利用分波分解分析

$$a_j(s) \equiv \frac{1}{32\pi}\int_{-1}^{1} \mathrm{d}\cos\theta P_j(\cos\theta)\, \mathcal{A}_s(s,\cos\theta). \tag{78}$$

The partial wave amplitudes encode the dependence of the spin j part of the amplitude on the center-of-mass energy s . The process (76) is then described by two nonzero partial wave amplitudes that capture the dependence of the amplitude on the spin-zero and spin-two parts of the graviton propagator (77),

> 分波振幅编码了振幅中自旋 j 部分对质心系能量 s 的依赖关系。过程 (76) 因此由两个非零分波振幅描述，它们分别捕捉了振幅对引力子传播子 (77) 自旋零部分和自旋二部分的依赖，

$$a_0(s) = \frac{G_N}{12} s^2 G_0(s), \; a_2(s) = -\frac{G_N}{60} s^2 G_2(s). \tag{79}$$

The analysis of the process $\phi\chi \to \phi\chi$ is more complicated. In this case, the amplitude also receives contributions from the t and u channel. The pole in the t -channel diagram leads to a divergence of the amplitude in the forward scattering limit. As a consequence, this process cannot be analyzed by a partial wave decomposition. However, this divergence is an IR effect and related to the massless nature of the graviton. Hence, its status is different from the UV divergences in the focus of our discussion.

> 对过程 $\phi\chi \to \phi\chi$ 的分析更为复杂。这种情况下，振幅还会收到来自 t 通道和 u 通道的贡献。t 通道图中的极点会导致前向散射极限下振幅发散，因此该过程无法通过分波分解分析。但该发散是红外效应，和引力子的无质量性质相关，因此它的性质和我们本文讨论重点的紫外发散不同。

General Relativity

广义相对论

Let us return to the case $\phi\phi \to \chi\chi$. The s -channel amplitude for this process in General Relativity (GR) is

> 我们回到 $\phi\phi \to \chi\chi$ 的情况。该过程在广义相对论 (GR) 中的 s 道振幅为

$$\mathcal{A}_s^{\phi\phi\to\chi\chi,\mathrm{GR}} = \frac{4\pi G_N}{3}\left(s - \frac{t^2 - 4tu + u^2}{s}\right). \tag{80}$$

The partial wave amplitudes describing tree-level scattering are obtained from (79) by setting the form factors to zero. In this way, one recovers the well-known result

> 描述树级散射的分波振幅可由式 (79) 将形状因子置零得到，由此可以重新得到广为人知的结果:

$$a_0^{\mathrm{GR}}(s) = \frac{G_N}{12} s, \; a_2^{\mathrm{GR}}(s) = -\frac{G_N}{60} s. \tag{81}$$

As their characteristic feature, these partial wave amplitudes grow without bound when the center-of-mass energy approaches infinity. Figure 2 illustrates the analytic properties of $a_2(s)$.

> 这类分波振幅的典型特征是，当质心系能量趋于无穷时，振幅会无限制增长。图 2 展示了 $a_2(s)$ 的解析性质。

The growth of the amplitude can already be deduced based on dimensional analysis. The partial wave amplitudes must be dimensionless. The topology of the tree-level diagram furthermore implies that the result must be proportional to G_N . In order to compensate the negative mass dimension of Newton’s constant, the amplitude has to come with positive powers of the center-of-mass energy. This argument readily extends to loop corrections appearing at higher orders in perturbation theory. These terms have to come with higher powers of the dimensionless quantity $G_N s$, aggravating the divergence. While this argument does not exclude that going beyond perturbation theory and performing a resummation of the series could lead to a well-defined UV behavior of the amplitude, it shows that treating graviton-mediated scattering within GR leads to unphysical divergences at high energy, at least at the perturbative level.

振幅的增长可以通过量纲分析直接推导得出: 分波振幅必须是无量纲的；此外，树级图的拓扑结构意味着结果必须正比于 G_N 。为了抵消牛顿常数的负质量量纲，振幅必须带有质心系能量的正幂次。这一论证可以直接推广到微扰论高阶出现的圈修正。这些项必然带有无量纲量 $G_N s$ 的更高次幂，会加剧发散。该论证虽没有排除非微扰处理、对级数进行重求和后振幅可能得到良好定义的紫外行为，但它表明，至少在微扰层面，在广义相对论框架下处理引力子介导的散射会在高能区出现非物理发散。

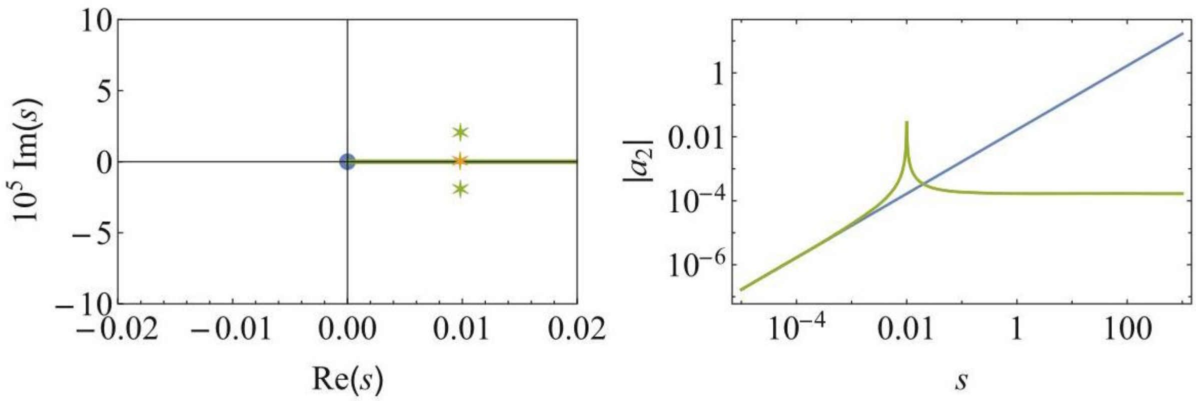

Fig. 2 Illustration of the graviton-mediated scattering process $\phi\phi \to \chi\chi$ described within quadratic gravity. The analytic properties of the spin-two propagator are displayed in the left panel. At tree level, $G_2\left(p^2\right)$ exhibits a massless pole (blue dot) and a ghost pole (orange star). Including the one-loop correction, the latter splits into a pair of complex-conjugate poles (green stars) and a branch cut (green line). The partial wave amplitude $a_2(s)$ is shown in the right panel. For center-of-mass energies below the ghost pole, the amplitude follows the one found in GR (blue line). At $s \approx (m_2)^2$, the amplitude for quadratic gravity exhibits a resonance. At energies larger than the ghost pole, the tree-level amplitude approaches a constant, while the inclusion of the one-loop correction to the propagator (green line) leads to a logarithmic fall-off. The latter reflects that quadratic gravity is asymptotically free. The illustration is obtained for $(m_2)^2 = 0.01, \varepsilon = 10^{-7}$, and $N_{\text{eff}} = 23/6$. For the selected parameter values, the tree-level and the one-loop amplitudes are virtually indistinguishable, showing that one ghost pole (orange) and the combination of two complex-conjugate poles and a branch cut (green) lead to very similar amplitudes

图 2 二次引力中描述的引力子介导散射过程 $\phi\phi \to \chi\chi$ 示意图。左图展示了自旋 2 传播子的解析性质: 树级层面，$G_2(p^2)$ 存在一个质量极点 (蓝色圆点) 和一个鬼极点 (橙色星形)。计入单圈修正后，鬼极点分裂为一对复共轭极点 (绿色星形) 和一个分支切割 (绿色线)。右图展示了分波振幅 $a_2(s)$ 。当质心系能量低于鬼极点质量时，振幅符合广义相对论中得到的结果 (蓝色线)；在 $s \approx (m_2)^2$ 处，二次引力的振幅出现共振。能量高于鬼极点质量时，树级振幅趋于常数，而对传播子计入单圈修正后 (绿色线) 振幅呈对数衰减，后者反映出二次引力是渐近自由的。本示意图在 $(m_2)^2 = 0.01, \varepsilon = 10^{-7}$ 和 $N_{\text{eff}} = 23/6$ 下绘制得到。对于所选参数值，树级振幅和单圈振幅几乎不可区分，说明单个鬼极点 (橙色) 与一对复共轭极点加一个分支切割的组合 (绿色) 给出的振幅非常相似

For the subsequent discussion, it is useful to extend this analysis to the process $\phi\chi \to \phi\chi$. This amplitude is obtained from (80) by crossing symmetry, $s \leftrightarrow t$, and reads

对于后续讨论，将分析拓展到过程 $\phi\chi \to \phi\chi$ 会很有帮助。该振幅可由式 (80) 通过交叉对称性 $s \leftrightarrow t$ 得到，形式为

$$\mathcal{A}_t^{\phi\chi\to\phi\chi,\text{GR}} = 8\pi G_N \frac{su}{t} = -8\pi G_N \frac{s(s+t)}{t}. \tag{82}$$

For forward scattering, $s \to \infty$ while keeping t fixed, this amplitude diverges quadratically. This exhibits two additional difficulties in analyzing gravity-mediated scattering amplitudes within GR. Firstly, the divergence in the forward scattering limit makes it difficult to apply the optical theorem. Secondly, the amplitude is not bounded in the Jin-Martin sense [63], i.e., it violates the constraint (73).

对于前向散射，在 $s \to \infty$ 趋近于零、保持 t 固定的情况下，该振幅是二次发散的。这体现出在广义相对论框架下分析引力子介导散射存在两个额外难点: 第一，前向散射极限下的发散使得光学定理难以应用；第二，振幅不满足金-马丁有界性条件 [63]，即违反了约束条件 (73)。

The general discussion of section ”Amplitudes from the Quantum Effective Action” then suggests that form factors can provide the crucial contributions that tame these divergences and render the amplitude finite. It is instructive to understand the underlying mechanisms based on two specific sets of form factors: the ones obtained in the framework of quadratic gravity [64] and the tanh model investigated in [25, 27] .

“量子有效作用量给出的振幅”章节的一般性讨论指出，形状因子可以提供关键贡献，驯服这些发散并使振幅变为有限。通过两组特定的形状因子来理解背后的机制很有启发性: 一组是在二次引力框架下得到的形状因子 [64]，另一组是 [25, 27] 中研究的双曲正切模型。

Quadratic Gravity

二次引力

Before moving toward the discussion of asymptotically safe amplitudes, it is instructive to take a short detour and review the prototype of the graviton-mediated asymptotically free scattering amplitudes realized in the framework of quadratic gravity (QG). In the language of form factors, (16), the bare action of this theory is obtained by taking $f_{CC}(x)$ and $f_{RR}(x)$ to be constant,

在讨论渐近安全散射振幅之前，我们不妨先绕一小段路，回顾一下在二次引力 (QG) 框架中实现的、由引力子介导的渐近自由散射振幅的原型。用形状因子的语言来说，该理论的裸作用量是通过令 $f_{CC}(x)$ 和 $f_{RR}(x)$ 为常数得到的，即式 (16)，

$$f_{CC}^{QG,\,\text{bare}} = -\frac{1}{\left(m_2^{\text{bare}}\right)^2}, \; f_{RR}^{QG,\,\text{bare}} = -\frac{1}{\left(m_0^{\text{bare}}\right)^2}. \tag{83}$$

Here m_0^{bare} and m_2^{bare} are the bare masses of the additional massive degrees of freedom accompanying the massless graviton familiar from GR. The presence of these additional degrees of freedom is readily seen by plugging (83) into (77) and investigating the scalar part of the tree-level propagators. Despite being a negative norm state, it has been argued that the ghost does not lead to a violation of unitarity, since its short lifetime implies that it is not an asymptotic state. In the modern interpretation due to Donoghue et al. [65] and Anselmi et al. [66,67], this additional degree of freedom has the status of a Merlin mode, moving backward in time, thereby violating causality at Planckian time scales. It is tempting to speculate that such an effect is the manifestation of quantum fluctuations in the light cone structure of spacetime at the level of the effective action [68].

此处 m_0^{bare} 和 m_2^{bare} 是伴随广义相对论中广为人知的无质量引力子存在的额外有质量自由度的裸质量。将式 (83) 代入式 (77)，研究树级传播子的标量部分，就能很容易看出这些额外自由度的存在。尽管该鬼场是负范数态，但已有研究指出它不会破坏幺正性，因为它的寿命极短，本身并不是渐近态。在 Donoghue 等人 [65] 和 Anselmi 等人 [66,67] 的现代诠释中，这个额外自由度具有默林模式的性质——它在时间中逆行，因此会在普朗克时间尺度上破坏因果性。我们很容易推测，这种效应正是时空光锥结构的量子涨落在有效作用量层面的体现 [68]。

The spin-two propagator takes the suggestive form

自旋 2 传播子具有如下启发性形式

$$G_2^{\text{tree}}\left(p^2\right) = \frac{1}{p^2} - \frac{1}{p^2 - \left(m_2^{\text{bare}}\right)^2}. \tag{84}$$

Thus the massive degree of freedom is a ghost, coming with a ”wrong sign” in the propagator. The analytic structure of the propagator and the resulting partial wave amplitude a_2 are illustrated in orange in Fig. 2. This shows that the ghost mode tames the growth of the amplitude at center-of-mass energies exceeding the pole mass. As a consequence, $\lim_{s\to\infty} a_2(s)$ is finite. The ghost thereby acts like a Pauli-Villars regulator, curing the UV divergence in the amplitude.

因此这个有质量自由度是一个鬼场，在传播子中带有“错误符号”。传播子的解析结构以及由此得到的分波振幅 a_2 已在图 2 中用橙色标出。这表明，当质心能量超过极点质量时，鬼模会抑制振幅的增长。结果就是 $\lim_{s\to\infty} a_2(s)$ 是有限的。鬼场在此起到泡利-维拉斯调节器的作用，解决了振幅中的紫外发散问题。

Upon including the one-loop self-energy corrections to (84), the scalar part of the propagator receives logarithmic corrections [69],

对式 (84) 加入单圈自能修正后，传播子的标量部分会得到对数修正 [69],

$$\left(G_2^{1-\mathrm{loop}}\left(p^2\right)\right)^{-1}=p^2+i\varepsilon-\frac{p^4}{m_2^2}-\frac{G_N}{20\pi}p^4N_{\mathrm{eff}}\log\left(\frac{-p^2-i\varepsilon}{\mu^2}\right). \tag{85}$$

Here μ^2 is an energy scale making the argument of the logarithm dimensionless, and $N_{\mathrm{eff}} = N_V + N_F/4 + N_S/6 + 21/6$ counts the number of light degrees of freedom comprising N_V vectors, N_F fermions, and N_S scalars. In our example with two massless scalar fields $N_{\mathrm{eff}} = 23/6$.

此处 μ^2 是使对数的自变量无量纲化的能标，$N_{\mathrm{eff}} = N_V + N_F/4 + N_S/6 + 21/6$ 对轻自由度的数目进行计数，其中包含 N_V 个矢量、N_F 个费米子和 N_S 个标量。在我们的例子中存在两个无质量标量场 $N_{\mathrm{eff}} = 23/6$ 。

The one-loop correction affects the analytic structure of the propagator, converting the ghost pole into a pair of complex-conjugate poles and a branch cut. This is illustrated in green in the left diagram of Fig. 2. Moreover, as shown in the right diagram of Fig. 2, the amplitude now decreases logarithmically for $s \gg m_2^2$. This reflects that quadratic gravity is asymptotically free. This conclusion holds modulo the caveat that the form factors related to non-minimal gravity-matter interactions (75), which have not been included in the analysis, do not modify the fall-off of the amplitudes in the UV.

单圈修正会改变传播子的解析结构，将鬼极点转化为一对复共轭极点和一个分支割线，这一变化已在图 2 左图中用绿色标出。此外，如图 2 右图所示，对于 $s \gg m_2^2$ ，振幅现在随对数形式衰减，这反映出二次引力是渐近自由的。该结论成立的前提是: 未被纳入分析的、与非最小引力-物质相互作用相关的形状因子 (75) 不会改变振幅在紫外区的衰减特性。

While the last property indicates that quadratic gravity is not asymptotically safe in the sense envisioned by Weinberg, one can nevertheless draw some important lessons from this example. First, (77) allows to readily recast the quantum corrections to the propagator in a one-loop form factor,

虽然上述性质表明，按照温伯格提出的定义，二次引力并不是渐近安全理论，但我们仍能从这个例子中得到一些重要启示。首先，式 (77) 可以很方便地将传播子的量子修正改写为单圈形状因子的形式,

$$f_{CC}^{\mathrm{QG},1-\mathrm{loop}}(x)=-\frac{1}{m_2^2}-\frac{G_N}{20\pi}N_{\mathrm{eff}}\log\left(\frac{-x-i\varepsilon}{\mu^2}\right), \tag{86}$$

where the $i\varepsilon$ selects the correct branch of the logarithm. On this basis, we expect that the effective action contains form factors with a nontrivial momentum dependence and a specific analytic structure in terms of branch cuts and poles. They play a crucial role in understanding the stability properties of resonances and graviton bound states that determine the high-energy behavior of the theory. Owed to the long-range nature of gravity, the form factors will also contain nonlocal terms.

其中 $i\varepsilon$ 对应对数的正确分支。在此基础上，我们预期有效作用量包含具有非平庸动量依赖的形状因子，且这些形状因子在分支割线和极点方面具有特定的解析结构。它们对于理解决定理论高能行为的共振和引力子束缚态的稳定性至关重要。由于引力的长程性质，形状因子还会包含非局域项。

Second, the discussion illustrates the link between form factors and results obtained from standard perturbative quantization techniques. Denoting the dimensionless coupling multiplying the Weyl-squared term by $1/\xi^2$, the one-loop contribution (85) can be understood as the logarithmic running of this coupling with respect to the physical momentum scale. Generally, this running should be discriminated from the scale dependence of couplings arising from integrating out quantum fluctuations shell-by-shell in a Wilsonian RG approach. For example, Newton's coupling has a nontrivial scale dependence with respect to the Wilsonian coarse graining scale k [5, 70] . At the same time, the structure of the effective action dictates that G_N cannot be promoted to a form factor and therefore must be constant at the level of Γ . See [32] for a detailed discussion. At the level of the effective action, this running is captured by a momentum-dependent form factor. In the case of $1/\xi^2$, this is given by $f_{CC}(\Delta)$.

其次，本讨论阐明了形状因子与通过标准微扰量子化技术得到的结果之间的关联。将乘以外尔平方项的无量纲耦合记为 $1/\xi^2$ ，一阶贡献 (85) 可以理解为该耦合随物理动量标度的对数跑动。一般而言，这种跑动与威尔逊重整化群方法中逐壳积分掉量子涨落产生的耦合标度依赖需要区分开。例如，牛顿耦合相对于威尔逊粗粒化标度 k [5, 70] 存在非平凡标度依赖。同时，有效作用量的结构表明 G_N 不能推广为形状因子，因此在 Γ 层面必须保持为常数。详细讨论参见文献 [32]。在有效作用量层面，这种跑动由依赖动量的形状因子刻画。对于 $1/\xi^2$ 的情形，其形式由 $f_{CC}(\Delta)$ 给出。

The Tanh Model

双曲正切模型

We now discuss a model that realizes physical asymptotic safety in the manner originally proposed by Weinberg [4]. We stress at this point that this model does not arise from a first principle calculation but was constructed to illustrate how the quantum effective action formalism can efficiently include concepts such as Asymptotic Safety, unitarity, and causality. The model was constructed and discussed in [25].

我们现在讨论一个实现了温伯格最初提出的物理渐近安全的模型 [4]。在此我们强调，该模型并非来自第一性原理计算，其构建目的是说明量子有效作用量形式体系如何能高效纳入渐近安全、幺正性和因果性这类概念。该模型的构建与讨论见文献 [25]。

The model again considers the graviton-mediated scattering of two minimally coupled scalar fields ϕ and χ with the form factors associated with the non-minimal gravity-matter couplings set to zero, cf. (75). The nontrivial form factors are present in the gravitational sector; we choose

该模型同样研究两个最小耦合标量场 ϕ 和 χ 由引力子介导的散射，其中与非最小引力-物质耦合相关的形状因子设为零，参见 (75)。非平凡形状因子存在于引力部分；我们选取

$$f_{RR}(\square) = c_{RR} G_N \tanh(c_{RR} G_N \square), \quad f_{CC}(\square) = c_{CC} G_N \tanh(c_{CC} G_N \square) .$$

(87)

Here $c_{RR}, c_{CC} > 0$ are numerical parameters that control the scale where the nontrivial form factors become significant. Finally, by specifying the scalar four-point amplitude, we fix the four-point form factor $f_{\phi^2\chi^2}$. This gives the four-point amplitude

此处 $c_{RR}, c_{CC} > 0$ 是控制非平凡形状因子效应变得显著的能标的数值参数。最后，通过指定标量四点振幅，我们固定了四点形状因子 $f_{\phi^2\chi^2}$ 。由此得到四点振幅

$$\mathcal{A}_4^{\phi\chi} = f_{\phi^2\chi^2}\left(\frac{s}{2},\frac{t}{2},\frac{u}{2},\frac{u}{2},\frac{t}{2},\frac{s}{2}\right) + \text{sym}, \tag{88}$$

where ”sym” indicates symmetrization of the arguments of $f_{\phi^2\chi^2}$ due to the functional variation of the action. The four-point amplitude is effectively parameterized by a function of three arguments that we choose to be

其中“sym”表示由于作用量的泛函变分，对 $f_{\phi^2\chi^2}$ 的宗量进行对称化。四点振幅实际上由一个三元函数参数化，我们选取该函数为

$$\mathcal{A}_4^{\phi\phi\to\chi\chi} = g(s \mid t,u)\ g(a \mid x,y) = 4\pi G_N G_{CC}(a)\left(x^2+y^2\right) \tag{89}$$

$$f^{\text{ int}}\left(a^2+x^2+y^2\right).$$

Here G_{CC} , defined in (51), is fixed by the graviton form factor f_{CC} . The function $f^{\text{ int}}$ interpolates between zero and one and is chosen to be

此处 G_{CC} 由 (51) 定义，由引力子形状因子 f_{CC} 固定。函数 $f^{\text{ int}}$ 在 0 和 1 之间插值，我们选取其形式为

$$f^{\text{ int}}(x) = \frac{c_t {G_N}^2 x \tanh\left(c_t {G_N}^2 x\right)}{1 + c_t {G_N}^2 x \tanh\left(c_t {G_N}^2 x\right)}. \tag{90}$$

Here $c_t > 0$ is a dimensionless parameter that controls the scale where the four-point form factor becomes significant. Plugging in these form factors into (48), we obtain the scattering amplitude for the $\phi\phi \to \chi\chi$ process.

此处 $c_t > 0$ 是一个无量纲参数，控制四點形状因子效应变得显著的能标。将这些形状因子代入 (48)，我们得到 $\phi\phi \to \chi\chi$ 过程的散射振幅。

We analyze the properties of the s -channel amplitude via the partial wave decomposition. We find the same partial wave amplitudes as in (79). Both partial wave amplitudes share the same basic behavior; for brevity, we will therefore only discuss the partial wave amplitude a_2 . It is plotted in Fig. 3. From this plot, we see that for small center-of-mass energies s , the partial wave amplitude a_2 does not differ from the GR amplitude. However, for large energies, rather than growing linearly with s , the amplitude a_2 becomes constant, reaching the asymptotic values

我们通过分波分解分析 s 道振幅的性质。我们得到与 (79) 相同的分波振幅。两个分波振幅的基本行为一致；因此为简洁起见，我们仅讨论分波振幅 a_2 。它绘制于图 3 中。从图中可以看到，对于小质心系能量 s ，分波振幅 a_2 与广义相对论的结果没有差异。但在大能量下，振幅 a_2 并不会随 s 线性增长，而是趋于常数，达到渐近值

$$\lim_{s\to\infty} a_0(s) = \frac{1}{12}\frac{1}{c_{RR}}, \quad \lim_{s\to\infty} a_2(s) = -\frac{1}{60}\frac{1}{c_{CC}}. \tag{91}$$

This is in agreement with the expectation from unitarity that the partial wave amplitudes are bounded by one, $|a_j(s)| \leq 1$.

这符合幺正性的预期，即分波振幅以 1 为界，$|a_j(s)| \leq 1$。

In order to claim that a setting is asymptotically safe, it does not suffice that just one amplitude remains bounded: All scattering amplitudes that can be constructed from the field content under consideration must be free from unphysical divergences. For the gravity-mediated scalar scattering, this implies that also the amplitude for the process $\phi\chi \to \phi\chi$ must be finite. The consequences of this requirement have been analyzed in detail in [25], and we summarize the central insights obtained from this case. We start from $\mathcal{A}_s^{\phi\chi}$ and use the crossing symmetry $s \leftrightarrow t$. Taking the GR limit, the resulting amplitude is given by (82). For forward scattering, $s \to \infty$ while keeping t fixed, this amplitude diverges quadratically in s . Inserting the form factors (87) does not improve this behavior.

若要宣称一个框架是渐近安全的，仅保证单个振幅有界并不充分: 由所考虑的场内容构造出的所有散射振幅都必须不存在非物理发散。对于引力介导的标量散射，这意味着过程 $\phi\chi \to \phi\chi$ 的振幅也必须有限。该要求的后果已在文献 [25] 中详细分析，我们在此总结从这个案例中得到的核心结论。我们从 $\mathcal{A}_s^{\phi\chi}$ 出发，利用交叉对称性 $s \leftrightarrow t$。取广义相对论极限，得到的振幅由 (82) 给出。对于向前散射，在保持 t 固定的条件下令 $s \to \infty$，该振幅随 s 二次发散。代入形状因子 (87) 无法改善这一行为。

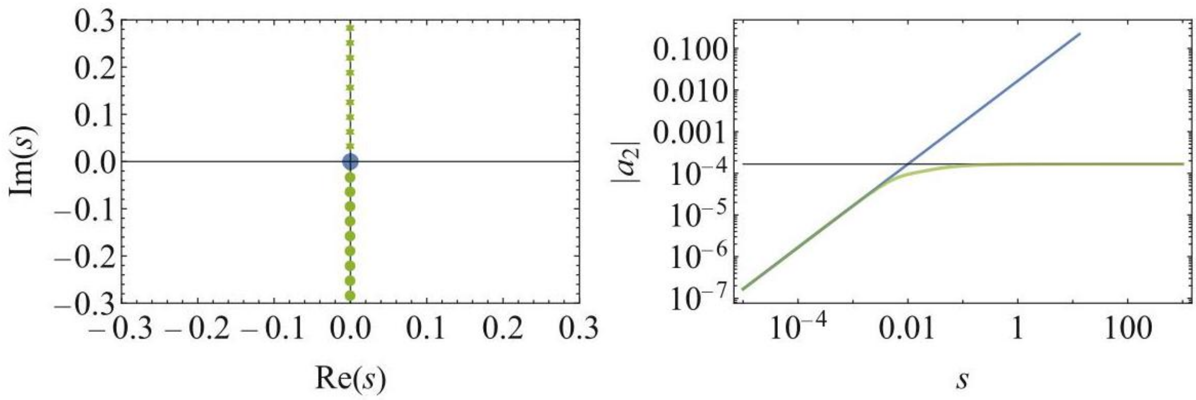

Fig. 3 Left panel: The analytic structure of the partial wave amplitude $a_2(s)$ for the tanh model. Green dots and stars mark poles at imaginary squared momentum with positive and negative residues, respectively. The blue dot gives the massless graviton pole from GR. Right panel: The partial wave amplitude $|a_2(s)|$ for the tanh model is shown as the green line, while the result of GR has been added as the blue line for reference. The black horizontal line shows the asymptotic value $\lim_{s\to\infty} |a_2(s)| = 1/6000$. The model parameters are $G_N = 1$ and $c_{CC} = 100$. At this point, we did not include the scalar self-interaction

图 3 左图: 双曲正切模型分波振幅 $a_2(s)$ 的解析结构。绿点和星形分别标记虚平方动量上 residue 为正和负的极点。蓝点对应广义相对论的无质量引力子极点。右图: 双曲正切模型的分波振幅 $|a_2(s)|$ 以绿线绘制，广义相对论结果以蓝线标出作为参考，黑色水平线标记渐近值 $\lim_{s\to\infty}|a_2(s)| = 1/6000$ 。模型参数为 $G_N = 1$ 和 $c_{CC} = 100$ ，此处未包含标量自相互作用

At this point, the four-point interaction becomes crucial. The four-point amplitude is readily derived from (89) by using crossing symmetry; we obtain

至此，四点相互作用变得至关重要。利用交叉对称性，可从式 (89) 直接推导出四点振幅，我们得到

$$\mathcal{A}_4^{\phi\chi\to\phi\chi} = g(t \mid s, u)\,. \tag{92}$$

The total scattering amplitude $\mathcal{A}^{\phi\chi\to\phi\chi} = \mathcal{A}_t^{\phi\chi} + \mathcal{A}_4^{\phi\chi\to\phi\chi}$ is shown in the right panel of Fig. 4. Here, we see that the total amplitude becomes scale-free in the forward scattering limit. Therefore, the interplay between the graviton propagator and the four-point interaction ensures the boundedness of the total scattering amplitude. In addition, this does not spoil the finiteness properties of the amplitude in the $\phi\phi \to \chi\chi$ process (left panel in Fig. 4).

总散射振幅 $\mathcal{A}^{\phi\chi\to\phi\chi} = \mathcal{A}_t^{\phi\chi} + \mathcal{A}_4^{\phi\chi\to\phi\chi}$ 如图 4 右图所示。可见在前向散射极限下，总振幅变为无标度形式。因此引力子传播子与四点相互作用的共同作用保证了总散射振幅的有界性。此外，这不会破坏 $\phi\phi \to \chi\chi$ 过程中振幅的有限性 (图 4 左图)。

At first sight, the necessary tuning of the matter self-interactions based on properties of the graviton propagator may appear artificial. From the perspective of Asymptotic Safety, such relations are not unexpected though: The UV behavior of an asymptotically safe theory is governed by an interacting fixed point of the Wilsonian RG. The enhanced symmetry, i.e., the quantum scale invariance originating from the fixed point, should then manifest itself in such a way that all scattering amplitudes remain bounded. Thus it is not inconceivable that the seemingly ad hoc relations between form factors in the high-energy limit have a natural explanation in terms of deeper symmetry principles underlying the construction.

乍看之下，根据引力子传播子的性质对物质自相互作用做必要调谐似乎显得刻意。但从渐近安全的角度来看，这类关系并不出人意料: 渐近安全理论的紫外行为由威尔逊重整化群的相互作用不动点支配。由此产生的增强对称性，即量子标度不变性，应当会体现为所有散射振幅始终保持有界。因此，高能极限下这些看似人为规定的形状因子之间的关系，完全可以由构建该理论背后更深层的对称性原理自然解释。

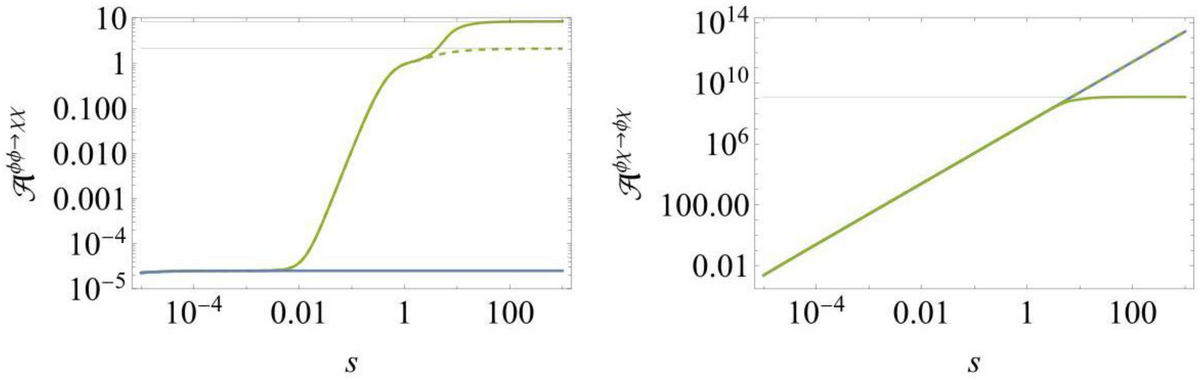

Fig. 4 The scattering amplitudes $\mathcal{A}^{\phi\phi\to\chi\chi}$ (left panel) and $\mathcal{A}^{\phi\chi\to\phi\chi}$ (right panel) in the forward scattering limit for GR (blue) and the tanh model (green). The solid line denotes the amplitude with self-interaction, while the dashed line depicts the s -channel diagram only. In these plots, $t = -10^{-6}{G_N}^{-1}$ is held fixed, with parameters $c_{RR} = 1, c_{CC} = 2$, and $c_t = 10^{-2}$. In both panels, asymptotes are denoted by thin gray lines

图 4 前向散射极限下广义相对论 (蓝色) 和双曲正切模型 (绿色) 的散射振幅 $\mathcal{A}^{\phi\phi\to\chi\chi}$ (左图) 与 $\mathcal{A}^{\phi\chi\to\phi\chi}$ (右图)。实线表示包含自相互作用的振幅，虚线仅表示 s 道图。图中 $t = -10^{-6}{G_N}^{-1}$ 保持固定，参数为 $c_{RR} = 1, c_{CC} = 2$ 和 $c_t = 10^{-2}$ 。两幅图中渐近线均以细灰线标记

Notably, this symmetry principle applies to the UV limit of the amplitudes only. In particular, we expect that there is a substantial freedom in how asymptotically safe amplitudes interpolate between their UV and IR parts. In the tanh model this freedom is reflected, for instance, in the choice of the interpolation function $f^{\text{int}}(x)$. In the language of the RG, this freedom is reflected by selecting a specific RG trajectory flowing away from the fixed point. The free parameters characterizing this choice (and parameterizing the UV critical surface of the fixed point) will also appear at the level of physical observables, e.g., determining the class of amplitudes being asymptotically safe with respect to this specific fixed point.

值得注意的是，该对称性原理仅适用于振幅的紫外极限。我们尤其认为，渐近安全振幅在紫外和红外区间之间的内插方式存在相当大的自由度。在双曲正切模型中，这种自由度例如体现在内插函数 $f^{\text{int}}(x)$ 的选择上。用重整化群的语言来说，这种自由度体现为选择一条从不动点流出的特定重整化群轨迹。表征该选择的自由参数 (也对不动点的紫外临界曲面做参数化) 同样会出现在物理可观测量层面，例如确定相对于该特定不动点渐近安全的振幅类。

Finally, we observe that the tanh model exhibits very peculiar properties with respect to causality. Focusing on the scalar part of the spin-two propagator, we find that its analytic structure encodes the contribution of the massless graviton supplemented by an infinite tower of unstable resonances coming with a mass and decay time set by Planckian scales. While the graviton sets a definite arrow of time, the resonances are distributed very symmetrically with respect to their causality properties. For each fixed resonance, there is exactly one positive and one negative energy mode which propagates forward and backward in time [71]. Hence the model is expected to violate microcausality on Planckian time scales. Whether this reflects a generic feature of asymptotically safe amplitudes is currently unknown.

最后，我们观察到双曲正切模型在因果性方面表现出非常特殊的性质。聚焦自旋二传播子的标量部分，我们发现其解析结构编码了无质量引力子的贡献，除此之外还包含一整串不稳定共振态，这些共振态的质量和衰变时间都由普朗克尺度决定。引力子确立了明确的时间箭头，而这些共振态的因果性质分布却十分对称。对每一个确定的共振态而言，恰好存在一个正能模和一个负能模，分别沿时间正向和反向传播 [71]。因此我们预期该模型会在普朗克时间尺度上违反微观因果性。目前这是否反映了渐近安全振幅的普遍特征尚不清楚。

Anomalous Dimensions Within the Form Factor Framework

形状因子框架内的反常维数

Let us finally make the connection between form factors and the anomalous dimension of the graviton propagator. Following [71], we define the anomalous dimension of a propagator $G_i\left(p^2\right)$ in the UV by

最后我们来建立形状因子与引力子传播子反常维数之间的联系。参考文献 [71]，我们对紫外区传播子 $G_i\left(p^2\right)$ 的反常维数定义如下

$$\eta_i^\infty \equiv \lim_{p^2\to\infty}\left(-p\frac{\partial}{\partial p}\ln\left(p^{-2}G_i^{-1}\left(p^2\right)\right)\right). \tag{93}$$

The relation between the form factors and the scalar parts of the propagators is given in (77) (with $\varepsilon = 0$). Now suppose that, for large momenta p^2 , the form factor under consideration follows the power law scaling

形状因子与传播子标量部分的关系已在式 (77) 中给出 (含 $\varepsilon = 0$)。现在假设，对于大动量 p^2 ，所讨论的形状因子满足幂律标度

$$f_i\left(p^2\right) \simeq \left(p^2\right)^a,\ a \in \mathbb{R}. \tag{94}$$

It is then straightforward to evaluate (93), yielding

接下来可以直接计算式 (93)，得到

$$\eta_i^\infty = \begin{cases} -2\left(1+a\right) a > -1, \\ 0\ a < -1. \end{cases} \tag{95}$$

We observe that, generically, there is a cap for the anomalous dimension. For $a < -1$ the scaling of the two-point function in the UV is no longer dominated by the form factor (94) but fixed by the Einstein-Hilbert contribution to the scalar part of the propagator. The fact that this transition among the leading behavior happens at $a = -1$ follows from the observation that the form factor contribution $f_i\left(p^2\right)$ is accompanied by a prefactor $\left(p^2\right)^2$ arising from the two spacetime curvature terms evaluated in the flat spacetime. Thus the Einstein-Hilbert term and the form factor exhibit the same UV scaling if $a = -1$. Hence, if the form factors are dictating the high-energy behavior of the propagator, the natural value for the UV anomalous dimensions obtained in the context of form factors is negative when compared to the p^2 -scaling of the propagator arising from the Einstein-Hilbert action.

我们发现，一般情况下反常维数存在上限。对于 $a<-1$ ，两点函数在紫外区的标度不再由形状因子 (94) 主导，而是由爱因斯坦-希尔伯特项对传播子标量部分的贡献固定。这种主导行为的转变发生在 $a=-1$ ，源于以下事实: 形状因子贡献 $f_i\left(p^2\right)$ 伴随一个前置因子 $\left(p^2\right)^2$ ，该因子来自平直时空下计算的两个时空曲率项。因此当 $a=-1$ 时，爱因斯坦-希尔伯特项与形状因子具有完全相同的紫外标度。因此，如果形状因子决定传播子的高能行为，那么在形状因子框架下得到的紫外反常维数，其自然值相比于爱因斯坦-希尔伯特作用量给出的传播子 p^2 标度为负。

However, this does not entail that form factors cannot accommodate positive values η_i^{∞} . Choosing

但这并不意味着形状因子无法容纳正的 η_i^{∞} 值。若取

$$f_i\left(p^2\right)=-\left(p^2\right)^{-1}+\left(p^2\right)^{\tilde{a}}(1+\text{ subleading }) \tag{96}$$

leads to

可得

$$\eta_i^{\infty}=-2(1+\tilde{a}),\ \tilde{a}\in\mathbb{R}, \tag{97}$$

which is positive for $\tilde{a}<-1$. This requires the precise cancellation between nonlocal form factor terms and the Einstein-Hilbert contribution: The first term in (96) has been precisely constructed in such a way that it eliminates the contribution of the latter in $G_i\left(p^2\right)$.

当 $\tilde{a}<-1$ 时该值为正。这要求非局域形状因子项与爱因斯坦-希尔伯特贡献精确相消: 式 (96) 中的第一项的构造方式恰好能消去后者在 $G_i\left(p^2\right)$ 处的贡献。

In principle, it is also possible to construct asymptotically safe amplitudes based on graviton propagators exhibiting a UV enhancement (manifested by $\eta_i^{\infty}>0$). In this case, the growth of the graviton propagator must be compensated by a suitable decay of the gravity-matter vertices similar to the cancellations induced by (96). Thus the interplay between the different building blocks for such amplitudes must be even more intricate than in the case where $\eta_i^{\infty}<0$.

原则上，我们也可以基于表现出紫外增强的引力子传播子 (体现为 $\eta_i^{\infty}>0$) 构造渐近安全散射振幅。在这种情况下，引力子传播子的增长必须通过引力-物质顶点的适当衰减来补偿，类似于式 (96) 诱导的抵消。因此，这类振幅不同构建模块之间的相互作用，会比 $\eta_i^{\infty}<0$ 的情况更加复杂。

Connection to Low-Energy Effective Field Theory

与低能有效场论的联系

Asymptotic Safety has the ambition to provide a description of gravity and matter valid on all momentum scales. The discussion of asymptotically safe amplitudes demonstrated that form factors are an essential element in the construction. Formulating Asymptotic Safety in terms of the effective action Γ also provides a direct link to low-energy effective field theory. This link allows to test the construction based on positivity bounds expected to hold for the low-energy theory in order to admit a UV completion. This discussion has a natural connection with the ”swampland program” investigated in the context of string theory [72]. At this point it should be stressed [73] that UV completions based on string theory and Asymptotic Safety may entail different definitions of the swampland since the space of effective field theories embeddable into string theory and admitting a UV completion through a Wilsonian RG fixed point may be different.

渐近安全有望提供对所有动量尺度都成立的引力与物质描述。对渐近安全散射振幅的讨论表明，形状因子是该结构的核心要素。以有效作用量 Γ 表述渐近安全，也直接建立了其与低能有效场论的关联。这一关联使得我们可以利用低能理论为容纳紫外完备性所须满足的正性界，对该构造进行检验。该讨论与弦理论框架下研究的“沼泽计划”存在自然关联 [72]。在此需要强调 [73]: 基于弦理论和渐近安全的紫外完备性对沼泽的定义可能并不相同，因为可嵌入弦理论的有效场论空间，与可通过威尔逊重整化群不动点实现紫外完备的有效场论空间可能并不一致。

Conceptually, the link between the effective action Γ of an asymptotically safe theory and its low-energy effective action is quite simple. Starting from Γ , one introduces a UV scale Λ_{UV} and express all couplings and momenta in terms of dimensionless quantities constructed from this scale. Subsequently, one expands Γ in (inverse powers of) the UV scale and truncates the expansion at a fixed order. In this way, one obtains the low-energy effective field theory capturing the physics at scales below Λ_{UV} . Technically, this expansion turns the form factor framework into a derivative expansion, restricting the retained interactions to the ones with a low mass dimension only. Provided that the form factor classification is carried out to sufficiently high order, the resulting expansion is consistent in the sense that it captures all interactions that appear in the effective field theory, potentially including quantum corrections.

从概念上讲，渐近安全理论的有效作用量 Γ 与其低能有效作用量之间的关联十分简单。从 Γ 出发，引入一个紫外标度 Λ_{UV} ，将所有耦合和动量都表示为由该标度构造的无量纲量。随后将 Γ 按紫外标度的负幂次展开，并在固定阶数截断展开。通过这种方式即可得到描述 Λ_{UV} 以下尺度物理的低能有效场论。从技术上讲，该展开将形状因子框架转化为导数展开，仅保留低质量维度的相互作用。只要形状因子分类做到足够高的阶数，最终得到的展开就是自洽的，能够涵盖有效场论中出现的所有相互作用，包括可能的量子修正。

The drawback of working in the derivative expansion is that it may lead to spurious poles in propagators and Ostrogradski-type instabilities [74] that may be absent in the full theory. Thus identifying the correct analytic structure of a propagator in this setting is far from trivial [75], and extra care has to be taken when interpreting the result. On the positive side, the derivative expansion directly connects to positivity bounds derived for the low-energy effective field theory. Typically, these bounds are obtained on the basis that the theory should come with a high-energy completion that is Lorentz invariant, local, and causal [76-78]. In particular, the optical theorem, stating that the imaginary part of the forward limit of an amplitude with the same initial and final particle content i , is given by

导数展开的缺点在于，它可能在传播子中引入假极点，还可能引发奥斯特罗格拉德斯基型不稳定性 [74]，而这些在完整理论中本不存在。因此，在该框架下确定传播子正确的解析结构绝非易事 [75]，解读结果时必须格外谨慎。从优势来看，导数展开可以直接对接为低能有效场论推导得到的正性界。这类界的一般推导前提是: 理论必须拥有洛伦兹不变、局域且因果的高能完备性 [76-78]。具体来说，光学定理指出，初态与末态粒子内容均为 i 的振幅，其向前极限的虚部等于

$$\operatorname{Im}\mathcal{A}_i(s,t)\big|_{t=0} = \frac{1}{2}\sum_f \int \mathrm{d}\Pi_f \left|\mathcal{A}_{i\to f}\right|^2 \geq 0. \tag{98}$$

Here $\mathcal{A}_{i\to f}$ is the amplitude for the process $i \to f$ with f denoting all possible admissible intermediate states, and $\int \mathrm{d}\Pi_f$ is the integral over the corresponding phase space. In the absence of negative norm states, all contributions on the righthand side are positive so that each intermediate state gives a positive contribution to the imaginary part of the forward scattering amplitude. While the optical theorem is very powerful for nongravitational theories, it is difficult to extend it to gravitational theories, see [10, 11] for a recent discussion. The reason is the t -channel pole,(82), whose residue grows faster than the Jin-Martin bound (73). Moreover, the pole at $t = 0$ and the associated branch point arising from graviton loops lead to severe complications when performing the analytic continuation from $t < 0$ to $t \geq 0$. This is essential in the derivation of positivity bounds.

此处 $\mathcal{A}_{i\to f}$ 是过程 $i \to f$ 的振幅，f 代表所有可能的容许中间态，$\int \mathrm{d}\Pi_f$ 是对应相空间上的积分。在没有负范数态的情况下，等式右侧所有贡献均为正，因此每个中间态对向前散射振幅的虚部都给出正贡献。尽管光学定理对非引力理论非常有效，但它很难推广到引力理论，近期相关讨论参见 [10, 11] 。原因在于 t 道极点即式 (82)，其留数增长速度快于金-马丁界即式 (73)。此外，$t = 0$ 处的极点以及引力子圈引发的对应分支点，给从 $t < 0$ 到 $t \geq 0$ 的解析延拓带来了严重的复杂性，而解析延拓是推导正性界的关键步骤。

Despite these complications, it is worthwhile to discuss a specific example of such bounds derived in the context of Quantum Electrodynamics (QED) minimally coupled to gravity. The corresponding action takes the form

尽管存在这些复杂性，讨论这类界的一个具体实例仍有价值: 该实例是在最小耦合引力的量子电动力学 (QED) 框架下推导得到的。对应的作用量形式为

$$S_{\text{QED}} = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{16\pi G_N}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\left(i\bar{\psi} + m\right)\psi + eA_\mu\bar{\psi}\gamma^\mu\psi\right].$$

(99)

The matter sector constitutes the prototype of a perturbatively renormalizable QFT. However, adding the minimal coupling to gravity makes the theory perturbatively non-renormalizable. Thus, (99) could be interpreted as an effective field theory. There is also evidence that gravity coupled to the matter content of QED admits UV completions via the Asymptotic Safety mechanism [79-81]. Note that several swampland conjectures also constrain the UV completion of QED coupled to gravity, see, e.g., [72, 82]. Our subsequent discussion of the low-energy bounds proposed for this system follows the exposition [9].

物质部分是微扰可重整量子场论的原型。但引入与引力的最小耦合后，该理论变为微扰不可重整。因此，式 (99) 可以被解释为一个有效场论。也有证据表明，耦合 QED 物质的引力可以通过渐近安全机制实现紫外完备 [79-81]。请注意，多个沼泽猜想也对耦合引力的 QED 的紫外完备性给出约束，例如参见 [72, 82]。我们后续对该系统低能界的讨论遵循文献 [9] 的阐述。

At energies below the mass of the electron, one may obtain an effective field theory capturing the dynamics of the photon and the graviton by integrating out the ”heavy” electron. This results in the Euler-Heisenberg action:

在能量低于电子质量的能区，人们可以通过积分掉“重”电子，得到一个描述光子和引力子动力学的有效场论。由此得到欧拉-海森伯作用量:

$$S_{\text{Eul-Heis}} = \int d^4x\sqrt{-g}\Big[-\frac{1}{16\pi G_N}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + a_1\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + a_2\left(F_{\mu\nu}\widetilde{F}^{\mu\nu}\right)^2$$

$$+b_1RF_{\mu\nu}F^{\mu\nu} + b_2R_{\mu\nu}F^{\mu\lambda}F^{\nu}{}_{\lambda} + b_3R_{\mu\nu\lambda\sigma}F^{\mu\nu}F^{\lambda\sigma}$$

$$+c_1R^2 + c_2R_{\mu\nu}R^{\mu\nu} + c_3R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma} + \cdots\Big].$$

(100)

This structure follows from the classification in section ”The Quantum Effective Action Including Form Factors” by eliminating all interaction monomials containing derivatives acting on the field strength tensor and setting the form factors to constants. The Wilson coefficients a_i, b_i , and c_i have been computed in [9, 83] but are not essential for the argument.

该结构可由“包含形状因子的量子有效作用量”一节中的分类得到: 去掉所有包含作用于场强张量的导数的相互作用单项式，并将形状因子设为常数。威尔逊系数 a_i, b_i 和 c_i 已在 [9, 83] 中计算得到，对本文论证而言并非核心。

The analysis of the physics consequences arising from (100) can be simplified by eliminating the Riemann-squared term through the Gauss-Bonnet identity and converting the Riemann tensor in the gravity-matter coupling into the Weyl tensor. Subsequently, the tree-level Einstein equations,

可以通过高斯-邦内恒等式消去黎曼平方项，并将引力-物质耦合中的黎曼张量转换为外尔张量，从而简化对 (100) 物理结果的分析。在此之后，利用树级爱因斯坦方程,

$$R_{\mu\upsilon} - \frac{1}{2}g_{\mu\upsilon}R = 8\pi G_N\left(\frac{1}{4}g_{\mu\upsilon}F_{\rho\sigma}F^{\rho\sigma} - F_{\mu\rho}F_{\upsilon}{}^{\rho}\right), \tag{101}$$

are used to eliminate the remaining curvature-squared terms and non-minimal gravity-matter couplings. This is equivalent to removing the inessential couplings with an Einstein-Hilbert starting point, as discussed earlier in section ”Field Redefinitions and Inessential Operators”. This procedure results in

消去剩余的曲率平方项和非最小引力-物质耦合。这等价于如前所述在“场重定义与非本质算符”一节中讨论的，从爱因斯坦-希尔伯特起点出发去掉非本质耦合。该过程最终得到

$$S_{\text{Eul-Heis},2} = \int d^4x\sqrt{-g}\Big[-\frac{1}{16\pi G_N}R - \frac{1}{4}F_{\mu\upsilon}F^{\mu\upsilon} + a_1'\left(F_{\mu\upsilon}F^{\mu\upsilon}\right)^2$$

$$+a_2'\left(F_{\mu\upsilon}\widetilde{F}^{\mu\upsilon}\right)^2 + b_3C_{\mu\upsilon\lambda\sigma}F^{\mu\upsilon}F^{\lambda\sigma} + \cdots\Big]$$

(102)

with the couplings being related by

其中耦合满足关系

$$a_1' = a_1 - 2\pi G_N b_2 + 4\pi G_N b_3 + \frac{1}{4}(8\pi G_N)^2 (c_2 + 4c_3)\,, \tag{103}$$

$$a_2' = a_2 - 2\pi G_N b_2 + 4\pi G_N b_3 + \frac{1}{4}(8\pi G_N)^2 (c_2 + 4c_3)\,.$$

The coefficients a_1' and a_2' can then be constrained based on general physics arguments. For instance, the absence of super-luminality in a certain electromagnetic configuration entails [9]

系数 a_1' 和 a_2' 可以通过一般物理原理得到约束。例如，特定电磁构型中不存在超光速性即给出 [9]

$$a_1' + a_2' \geq 0 \tag{104}$$

The same conclusion arises from analyzing the symmetrized scattering amplitude associated with light-by-light scattering,

相同结论可以通过分析光子-光子散射对应的对称化散射振幅得到,

$$\mathcal{A}_{\text{sym}} \equiv \mathcal{A}^{++--} + \mathcal{A}^{--++} + \mathcal{A}^{+--+} + \mathcal{A}^{-++-}. \tag{105}$$

Evaluating the general expression (54) for the specific couplings (100), one obtains in the forward scattering limit

将通式 (54) 代入特定耦合 (100) 计算，在前向散射极限下可得

$$\lim_{t\to 0} \mathcal{A}_{\text{sym}}(s,t) = -\frac{32\pi G_N s^2}{t} - 32\pi G_N s + 32\,(a_1' + a_2')\,s^2 + \mathcal{O}(t)\,. \tag{106}$$

The first term is the t -channel pole, signaling the divergence of the amplitude in the forward limit. Requiring that the coefficient appearing in front of the s^2 -term is positive again implies the bound (104).

第一项是 t 道极点，预示着振幅在前向极限下发散。要求 s^2 项前的系数为正同样会给出约束 (104)。

Based on unitarity, one can even argue a more stringent bound, suggesting that both coefficients be positive individually,

基于幺正性，我们甚至可以得到更严格的约束，即两个系数各自都必须为正,

$$a_1' \geq 0,\ a_2' \geq 0. \tag{107}$$

The underlying idea is that these effective couplings are generated by UV degrees of freedom that do not contain states of negative norm. The argument then develops along the following lines. To leading order, the coupling between the photon and these UV degrees of freedom is parameterized by

其核心思想是，这些有效耦合由不包含负范数态的紫外自由度产生。论证过程如下。领头阶下，光子与这些紫外自由度的耦合可以参数化为

$$F^{\mu\upsilon}F^{\rho\sigma}\chi_{\mu\upsilon\rho\sigma},\ F^{\mu\upsilon}\widetilde{F}^{\rho\sigma}\psi_{\mu\upsilon\rho\sigma}. \tag{108}$$

The fields χ and ψ are parity even and odd, respectively, and possess the same index symmetry as the Riemann tensor. Moreover, any couplings have been absorbed into these fields. In order to foster the analysis, they are decomposed into their trace and traceless parts, i.e.,

> 场 χ 和 ψ 分别是宇称偶和宇称奇，且具有和黎曼张量相同的指标对称性。此外，所有可能的耦合都已经被吸收到这些场中。为了便于分析，我们将它们分解为迹部分和无迹部分，即:

$$\chi_{\mu\nu\rho\sigma}=\chi^{(4)}_{\mu\nu\rho\sigma}+\frac{1}{4}\left(\eta_{\mu[\rho}\chi^{(2)}_{\sigma]\nu}-\eta_{\nu[\rho}\chi^{(2)}_{\sigma]\mu}\right)+\frac{1}{2}\chi^{(0)}\eta_{\mu[\rho}\eta_{\sigma]\nu}, \tag{109}$$

and similarly for ψ . The tensorial component fields $\chi^{(4)}_{\mu\upsilon\rho\sigma}$ and $\chi^{(2)}_{\mu\upsilon}$ are traceless by definition. One then stipulates that the propagators of the component fields admit a standard spectral representation

> 对 ψ 可做类似分解。张量分量场 $\chi^{(4)}_{\mu\upsilon\rho\sigma}$ 和 $\chi^{(2)}_{\mu\upsilon}$ 按定义是无迹的。随后我们假设分量场的传播子满足标准谱表示

$$\left\langle\chi^{(0)}(p)\,\chi^{(0)}(p')\right\rangle=i\delta^4(p+p')\int_0^\infty \mathrm{d}\mu^2\frac{\rho^{(0)}(\mu^2)}{p^2-\mu^2+i\varepsilon},$$

$$\left\langle\chi^{(2)}_{\mu\nu}(p)\,\chi^{(2)}_{\alpha\beta}(p')\right\rangle=i\delta^4(p+p')\int_0^\infty \mathrm{d}\mu^2\frac{\rho^{(2)}(\mu^2)}{p^2-\mu^2+i\varepsilon}\Pi_{\mu\nu\alpha\beta},$$

$$\left\langle\chi^{(4)}_{\mu\nu\rho\sigma}(p)\,\chi^{(4)}_{\alpha\beta\gamma\delta}(p')\right\rangle=i\delta^4(p+p')\int_0^\infty \mathrm{d}\mu^2\frac{\rho^{(4)}(\mu^2)}{p^2-\mu^2+i\varepsilon}\Pi_{\mu\nu\rho\sigma\alpha\beta\gamma\delta}.$$

(110)

The spectral densities $\rho^{(i)}(\mu^2)$ are supposed to be positive definite and capture arbitrary collections of single- and multi-particle states. The Π are constructed from the Minkowski metric $\eta_{\mu\nu}$ and four-momentum p_μ and carry the tensor structure of the correlators. A careful analysis of their properties reveals [9] that $\Pi_{\mu\upsilon\rho\sigma\alpha\beta\gamma\delta}$ necessarily involves negative norm states. On this basis, it is excluded, and the four-point interactions are assumed to be generated from $\chi^{(0)}$ and $\chi^{(2)}_{\mu\nu}$ only. Going to low energy, $p^2\to 0$, one then finds spectral representations of the effective couplings a_1' and a_2' . For instance

> 谱密度 $\rho^{(i)}(\mu^2)$ 应为正定，描述任意的单粒子态与多粒子态集合。Π 由闵氏度规 $\eta_{\mu\nu}$ 和四动量 p_μ 构造，携带关联函数的张量结构。对其性质的细致分析表明 [9]，$\Pi_{\mu\upsilon\rho\sigma\alpha\beta\gamma\delta}$ 必然包含负范数态。因此它被排除在外，四点相互作用被认为仅由 $\chi^{(0)}$ 和 $\chi^{(2)}_{\mu\nu}$ 产生。到低能区，$p^2\to 0$，我们可以得到有效耦合 a_1' 和 a_2' 的谱表示。例如

$$F^{\mu\upsilon}F^{\rho\sigma}\chi_{\mu\upsilon\rho\sigma}\to\frac{1}{12}\left(F_{\mu\upsilon}F^{\mu\upsilon}\right)^2\int_0^\infty \mathrm{d}\mu^2\frac{6\rho^{(0)}(\mu^2)+\rho^{(2)}(\mu^2)}{\mu^2+i\varepsilon}$$

(111)

$$+\frac{1}{8}\left(F_{\mu\nu}\widetilde{F}^{\mu\nu}\right)^2\int_0^\infty \mathrm{d}\mu^2\frac{\rho^{(2)}(\mu^2)}{\mu^2+i\varepsilon}.$$

The bounds (107) then arise from the positivity of the spectral integral. Notably, this is a weaker condition than demanding that $\rho^{(i)}(\mu^2)$ is a positive function. Therefore, the latter may not strictly be necessary to conclude that the couplings generated by the ultraviolet states are positive.

约束 (107) 即可由谱积分的正定性得到。值得注意的是，这比要求 $\rho^{(i)}(\mu^2)$ 是正函数的条件更弱。因此，要得出紫外态产生的耦合为正的结论，后者并非严格必要。

From the perspective of Asymptotic Safety, it is worthwhile to make the following observation. The argument leading to (107) tacitly assumes that the corresponding operators are absent in the ultraviolet theory. Owed to the nonvanishing gravitational interactions present at the underlying RG fixed point, it is expected that these interactions are already present in the fixed point action [84] since they are compatible with the symmetries of the kinetic term of the photon and therefore evade the non-renormalization condition [85]. In this light, it is conceivable that a positive spectral function of the ultraviolet contributions rather reflects itself in a monotonicity property of the Wilsonian RG flow rather than a positivity bound of the form (107) [86].

从渐近安全的视角来看，做出下述观察很有价值。推导出式 (107) 的论证默认假设了紫外理论中不存在对应算符。由于底层重整化群不动点处存在非零引力相互作用，我们预期这类相互作用本就存在于不动点作用量 [84] 中——因为它们与光子动能项的对称性相容，因此不满足非重整化条件 [85]。据此来看，紫外贡献的正谱函数更有可能体现为威尔逊重整化群流的单调性，而非式 (107) 形式的正性界 [86]。

Quite remarkably, the condition of causality also gives rise to constraints on the gravitational three-point vertex. Reference [8] analyzed these conditions for the case of a weakly coupled gravitational theory, concluding that certain higher-derivative corrections to the three-point vertex may lead to a time advance in a high-energy scattering process that can overwhelm the classical Shapiro time delay. In the context of quadratic gravity, these arguments have recently been refined in [87]. Moreover, bounds from four-graviton scattering at one-loop level have recently been discussed in [88]. It would be interesting to investigate to which extent form factors may alter these conclusions.

非常值得注意的是，因果性条件也会对引力三点顶点给出约束。文献 [8] 分析了弱耦合引力理论的这类约束，得出结论: 三点顶点的某些高阶导数修正可能会在高能散射过程中产生时间超前，其效应超过经典的夏皮罗时间延迟。在二次引力的框架下，这些论证近期在文献 [87] 中得到了改进。此外，单圈四引力子散射的界近期也在文献 [88] 中得到讨论。研究形状因子会在多大程度上改变这些结论会是很有意义的工作。

Form Factors from First Principles

第一性原理形状因子

So far, our discussion adopted a perspective that focused on the general form of a scattering amplitude in the presence of form factors. In general, any quantum theory of gravity is expected to make distinct predictions for the form factors. This fixes the properties of the amplitudes arising from the approach. In this section, we review the status of first principle computations of form factors within asymptotically safe quantum gravity. So far, two routes have been investigated to obtain non-perturbative form factors: functional RG methods on the one hand and the reconstruction from Monte Carlo data on the other. These techniques and the corresponding results are described in sections ”Form Factors from the Functional Renormalization Group” and ”Reconstruction from Monte Carlo Data”, respectively.

到目前为止，我们的讨论始终围绕存在形状因子时散射振幅的一般形式展开。一般而言，任何量子引力理论都应对形状因子给出不同的预言，这由此方法得到的振幅性质确定。本节我们回顾渐近安全量子引力框架下形状因子第一性原理计算的研究现状。到目前为止，研究者已经探究了得到非微扰形状因子的两条路径: 一方面是泛函重整化群方法，另一方面是从蒙特卡洛数据重构。这些技术和对应结果分别在“泛函重整化群导出形状因子”和“蒙特卡洛数据重构”两节中介绍。

Form Factors from the Functional Renormalization Group

来自泛函重整化群的形状因子

The first-principle construction of non-perturbative form factors based on functional RG methods predominantly employs the Wetterich equation [5, 13, 14] in Euclidean signature spacetimes. This equation realizes the idea of the Wilsonian RG, integrating out quantum fluctuations shell-by-shell in momentum space. Concretely, it encodes the dependence of the effective average action Γ_k on the coarse graining scale k :

基于泛函 RG 方法的非微扰形状因子第一性原理构造主要采用欧几里得符号时空下的韦特里希方程 [5, 13, 14] 。该方程实现了威尔逊 RG 的思想，即在动量空间中逐层积分掉量子涨落。具体而言，它编码了有效平均作用量 Γ_k 对粗粒化标度 k 的依赖关系:

$$k\partial_k\Gamma_k = \frac{1}{2}\,\mathrm{Tr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} k\partial_k R_k\right]. \tag{112}$$

Here $\Gamma_k^{(2)}$ denotes the second functional derivative of Γ_k with respect to the fluctuation fields, R_k is an IR regulator, equipping fluctuations with momenta $p^2 \lesssim k^2$ with a k -dependent mass term, and the trace contains an integral over loop momenta as well as a sum over fields. The interplay between the regulator terms on the right-hand side ensures that the trace is UV finite, so that the change of Γ_k is actually driven by integrating out fluctuations with momenta $p^2 \approx k^2$. In the context of gravity, the construction of (112) requires the introduction of an undetermined background metric $\bar{g}_{\mu\nu}$, so that the spacetime metric $g_{\mu\nu}$ can be decomposed into its background part and arbitrary fluctuations $h_{\mu\nu}$, e.g., through the linear split

此处 $\Gamma_k^{(2)}$ 表示 Γ_k 对涨落场的二阶泛函导数，R_k 是红外调节器，为动量为 $p^2 \lesssim k^2$ 的涨落引入一个依赖 k 的质量项，迹中包含圈动量积分以及对场的求和。方程右侧调节器项的相互作用保证了迹在紫外是有限的，因此 Γ_k 的变化实际上由积分掉动量为 $p^2 \approx k^2$ 的涨落驱动。在引力研究中，(112) 式的构造需要引入未定背景度规 $\bar{g}_{\mu\nu}$ ，因此时空度规 $g_{\mu\nu}$ 可以分解为背景部分和任意涨落 $h_{\mu\nu}$ ，例如通过线性分解

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}. \tag{113}$$

The background then allows us to construct a reference scale, discriminating UV from IR modes. More details on the Wetterich equation and its application to gravity can be found in the reviews [89-92] as well as in the introductory chapter of this volume [93].

背景由此可以用来构造参考标度，区分紫外模式和红外模式。关于韦特里希方程及其在引力中的应用的更多细节可以在综述文献 [89-92] 以及本卷的介绍章节 [93] 中找到。

A standard strategy for extracting non-perturbative information from (112) consists in making an ansatz for Γ_k (called truncation), supplying a closure condition for the right-hand side, and subsequently computing the flow of the k -dependent quantities retained by the truncation. At the level of form factors, this procedure generically leads to complicated nonlinear integro-differential equations that need to be solved numerically. At the technical level, it is useful to discriminate among computations determining form factors at the background level and the nontrivial momentum dependence in correlation functions built from the fluctuation fields. Owed to the presence of the gauge-fixing and regulator terms breaking the split symmetry (113), the results do not necessarily agree [94]. The former may then be reconstructed from the latter by solving additional split-Ward or Nielsen identities.

从 (112) 式中提取非微扰信息的标准策略是: 对 Γ_k 做一个近似假设 (称为截断)，为方程右侧提供封闭条件，随后计算截断保留的依赖 k 的量的重整化流。在形状因子层面，该过程一般会得到复杂的非线性积分微分方程，需要通过数值方法求解。在技术层面，区分两类计算是有必要的: 一类是在背景层面确定形状因子，另一类是涨落场构造关联函数中非平凡的动量依赖。由于规范固定项和调节器项破缺了 (113) 式的分解对称性，两类计算的结果并不一定一致 [94]。前者可以通过求解额外的分解沃德恒等式或尼尔森恒等式从后者重构得到。

Background Form Factors

背景形状因子

The computation of background form factors is closely linked to the classification of interaction monomials given in section ”The Quantum Effective Action Including Form Factors”. The truncation of Γ_k is obtained by promoting the form factors appearing in Γ to k -dependent functions. Using (112), one determines the flow of these functions with respect to the coarse graining scale, as well as their k -stationary forms at a RG fixed point. The evaluation of the trace can be done in a background-independent way employing (off-diagonal) heat kernel techniques [30, 95-98].

背景形状因子的计算与“包含形状因子的量子有效作用量”一节给出的相互作用单项式分类密切相关。通过将 Γ 中出现的形状因子推广为依赖于 k 的函数，可得到 Γ_k 的截断形式。利用式 (112)，我们可以确定这些函数关于粗粒化标度的流，以及它们在重整化群不动点处的 k 平稳形式。迹的计算可以采用背景无关的非对角热核技术完成 [30, 95-98]。

So far, these computations have mostly focused on the propagators and the gravitational form factors induced by free matter fields. The canonical example is the rederivation of the Polyakov action in two dimensions [99], which yields a nonlocal form factor contribution

到目前为止，这类计算大多集中在自由物质场诱导的传播子和引力形状因子上。典型例子是在二维中重新推导了 Polyakov 作用量 [99]，得到了非局域形状因子贡献

$$\Gamma^{\text{Pol}} = -\frac{1}{96\pi}\int \mathrm{d}^2 x\sqrt{g}R\frac{1}{\Box}R, \tag{114}$$

for a single scalar field. For the physical case of four dimensions, the low-energy form factors have been systematically investigated in [100-104], including some cosmological applications [105].

对应单个标量场。对于物理的四维情形，低能形状因子已在文献 [100-104] 中得到系统研究，其中还包含了一些宇宙学应用 [105]。

A first self-consistent computation of a gravitational form factor has been put forward in [106], where f_{CC} was computed within conformally reduced quantum gravity. Qualitatively, a very good fit for positive (Euclidean) momenta was given as

首个引力形状因子的自洽计算由文献 [106] 提出，该工作在共形约化量子引力框架下计算了 f_{CC} 。定性来看，该结果对正 (欧几里得) 动量给出了非常好的拟合，形式为

$$f_{CC}(x) \approx f_\infty + \frac{\rho}{\frac{\rho}{\kappa}+x}, \tag{115}$$

for numerical parameters f_∞, ρ, κ . Incidentally, this is very similar to what one finds for the propagator from RG improvement [107].

其中数值参数为 f_∞, ρ, κ 。巧合的是，这与重整化群改进得到的传播子结果非常相似 [107]。

Following up on [74], a general toolbox to compute form factors from first principles was provided in [7]. There, also $f_{\phi\phi}$ was computed, but almost no deviation from a free scalar field was found. A similar computation, but in a different gauge, was performed in [108], where the deviations from a free propagator vanished identically for a massless scalar.

继文献 [74] 之后，文献 [7] 给出了从第一性原理计算形状因子的通用工具包，该工作也计算了 $f_{\phi\phi}$ ，但几乎没有发现偏离自由标量场的结果。文献 [108] 在不同规范下进行了类似计算，发现对于无质量标量场，与自由传播子的偏差严格为零。

Form Factors Related to Fluctuation Fields

与涨落场相关的形状因子

In contrast to the background form factor computations, the fluctuation approach (see [109-112] for early works) typically adopts a flat Euclidean background and organizes the effective average action in terms of n -point functions built from the fluctuation field,

与背景形状因子计算不同，涨落方法 (早期工作参见 [109-112]) 通常采用平坦欧几里得背景，并将有效平均作用量按由涨落场构造的 n 点函数整理，

$$\Gamma_k = \sum_n \int \prod_{i=1}^{n} \frac{d^4 p_i}{(2\pi)^4}\delta^4\left(\sum_i p_i\right)\Gamma^{(n)}_{\mu_1 v_1\cdots\mu_n v_n}(k;p_i)\,h_{\mu_1 v_1}(p_1)\cdots h_{\mu_n v_n}(p_n).$$

(116)

The $\Gamma^{(n)}$ depends on all momenta of the fluctuation fields and carries nontrivial tensor structures. For instance, picking $\Gamma_k^{(2)}$ and focusing on the tensor structure associated with the transverse-traceless part of the graviton fluctuation allow us to determine the momentum dependence of the graviton two-point function. Owed to the flat background, the evaluation of the trace can be done using standard momentum-space techniques. This technical advantage has allowed to resolve much more information. Concretely, the full graviton two-point function and parts of the graviton three- and four-point functions have been investigated, corresponding to form factors with two, three, and four powers of the curvature. Beyond that, several gravity-matter form factors have been calculated.

> $\Gamma^{(n)}$ 依赖于涨落场的所有动量，具有非平凡张量结构。例如，选取 $\Gamma_k^{(2)}$ 并关注与引力子涨落的横向无迹部分关联的张量结构，即可确定引力子两点函数的动量依赖。由于背景是平坦的，我们可以用标准动量空间技术完成迹的计算。这一技术优势让我们能够得到多得多的信息。具体而言，研究者已经研究了完整的引力子两点函数，以及引力子三点、四点函数的部分结构，分别对应带有二次、三次、四次曲率幂次的形状因子。除此之外，若干引力-物质形状因子也已经完成计算。

The first work investigating the propagator of the graviton (and its attached Faddeev-Popov ghost) was [113]. In this chapter, the propagators were computed for the transverse-traceless component (transverse component for the ghost) only. A comparison with a derivative expansion showed that the latter is rather unstable at the lowest order, and one should implement either a constant anomalous dimension or a bi-local approximation.

> 研究引力子传播子 (及其附带的法捷耶夫-波波夫鬼) 的第一项工作是文献 [113]，该工作仅计算了横向无迹分量 (鬼场对应横向分量) 的传播子。与导数展开的对比表明，导数展开在最低阶相当不稳定，应当要么引入常数反常维数，要么采用双局域近似。

The resolution of all components of the graviton and ghost propagator was achieved only recently [104]. This revealed that the different graviton modes exhibit significant qualitative differences in their momentum dependence. On the other hand, the transverse and longitudinal modes of the ghost share a very similar propagator. Some general asymptotic relations of propagators have been derived, for example, that for large momenta the two propagators of the ghost modes agree identically. Conceptually, [104] elucidated the relation between momentum-dependent correlation functions and the form factors.

> 直到最近，文献 [104] 才完整求解了引力子和鬼场传播子的所有分量。结果显示，不同引力子模式的动量依赖存在显著的定性差异。另一方面，鬼场的横向模式和纵向模式的传播子非常相似。研究者还推导出了传播子的若干普适渐近关系，例如大动量下鬼场两种模式的传播子完全一致。从概念层面，文献 [104] 阐明了动量依赖关联函数与形状因子之间的关系。

The first investigation of the momentum dependence of the three-graviton vertex [114] also introduced a nontrivial condition, dubbed momentum locality, on the asymptotic scaling of the flow of correlation functions. Momentum locality is inherently related to a well-defined coarse graining. Some, but not all graviton correlation functions, possess this property [94,104,114]. For example, the transverse-traceless part of the propagator as well as a specific projection of the three-graviton vertex displays momentum locality, whereas the spin-zero part of the propagator does not.

第一项研究三引力子顶点动量依赖的工作 [114] 还对关联函数流的渐近标度提出了一个非平凡条件，称为动量局域性。动量局域性本身和定义良好的粗粒化直接相关。部分 (而非全部) 引力子关联函数满足这一性质 [94,104,114]，例如传播子的横向无迹部分以及三引力子顶点的特定投影满足动量局域性，而传播子的零自旋部分不满足该性质。

The investigation of the four-graviton vertex [115] provided indications for an apparent convergence of the vertex expansion regarding the stability of the asymptotically safe fixed point. It also concluded that the generation of the operator $R_{\mu\upsilon}R^{\mu\upsilon}$ is suppressed compared with the generation of R^2 .

对四引力子顶点的研究 [115] 为渐近安全固定点稳定性相关的顶点展开收敛性提供了支持。该研究还得出结论: 算符 $R_{\mu\upsilon}R^{\mu\upsilon}$ 的产生相比于 R^2 的产生是被压低的。

As a step beyond the form factor expansion about a flat background, [116] resolved the momentum and (Ricci scalar) curvature dependence of the graviton propagator. Along the lines of the discussion in section ”Going Beyond Flat Spacetime”, this corresponds to a form factor expansion about a manifold with constant Ricci scalar which is necessary to compute exact scattering amplitudes on de Sitter and anti-de Sitter spaces [60,61]. Intriguingly, some of the couplings turn out to be approximately independent of the curvature.

作为超出平坦背景形状因子展开的一步，文献 [116] 求解了引力子传播子的动量依赖和 (里奇标量) 曲率依赖。按照“超出平坦时空”一节中的讨论，这对应于常里奇标量流形上的形状因子展开，而这种展开是计算德西特和反德西特空间上精确散射振幅的必要基础 [60,61]。有趣的是，部分耦合近似与曲率无关。

The most recent addition to the discussion of momentum dependence in quantum gravity is the computation of the graviton spectral function, from both Euclidean [117] and Lorentzian [118] computations. This work represents a crucial step on the path to computing asymptotically safe scattering amplitudes, since all previous computations were carried out with Euclidean signature, whereas amplitudes probe genuine Lorentzian momentum configurations. This is particularly so for massless fields, where the on-shell condition $p^2 = 0$ in Euclidean signature implies a completely vanishing momentum vector. One key result of these investigations is that the (transverse-traceless) fluctuation graviton spectral function exists and is positive. A second key insight is that also non-perturbatively, there is only a single pole at vanishing mass, so that no additional ghosts appear in the spectrum.

量子引力动量依赖研究的最新进展是引力子谱函数的计算，分别完成了欧几里得 [117] 和洛伦兹 [118] 下的计算。这项工作是计算渐近安全散射振幅过程中的关键一步，因为此前所有计算都在欧几里得号差下完成, 而散射振幅描述的是真正的洛伦兹动量构型。对于无质量场这一点尤其明显, 因为在欧几里得号差下，在壳条件 $p^2 = 0$ 意味着动量矢量完全为零。这些研究的一个核心结果是:(横向无迹) 涨落引力子的谱函数存在且为正。第二个核心结论是: 即使在非微扰层面，仅在零质量处存在一个极点，因此谱中不会出现额外的鬼场。

Coming to gravity-matter systems, an interesting relation for some gravitational diagrams contributing to the gluon propagator was found in [119]. More concisely, it was found that in the weak-gravity limit the gauge coupling does not receive contributions from the gravitational sector. A more detailed study of the two- and three-point functions of this system was put forward in [120]. The scalar and fermionic propagators coupled to gravity were studied in [108]. In a particular gauge, there is no quantum correction to the propagator of a massless scalar. In [121], the concept of effective universality was introduced and found to be present approximately at the asymptotically safe fixed point, see also [122]. The idea is that there are nontrivial relations related to diffeomorphism invariance that different vertices have to satisfy. In [25] it was remarked that such relations must be present for scattering amplitudes to possess a well-behaved high-energy limit. The momentum dependence of the graviton-fermion-fermion vertex was investigated in [123], where indications were found that such a non-minimal coupling plays a sub-leading role. Finally, [124] extended the study of momentum- and curvature-dependent propagators to a gravity-scalar system.

对于引力-物质系统，文献 [119] 发现了一个有趣的关系，适用于对胶子传播子有贡献的部分引力图。更简洁地说，研究发现在弱引力极限下，规范耦合不会受到引力 sector 的贡献。文献 [120] 对该系统的两点和三点函数开展了更详细的研究。文献 [108] 研究了与引力耦合的标量和费米子传播子。在特定规范下，无质量标量的传播子不存在量子修正。文献 [121] 引入了有效普适性的概念，并发现在渐近安全不动点处近似存在该性质，另见文献 [122]。其核心观点是，不同顶角必须满足一些与微分同胚不变性相关的非平凡关系。文献 [25] 指出，为了让散射振幅具有表现良好的高能极限，这类关系必须成立。文献 [123] 研究了引力子-费米子-费米子顶角的动量依赖，发现结果表明这种非最小耦合起到次领头阶作用。最后，文献 [124] 将动量和曲率依赖传播子的研究推广到了引力-标量系统。

Reconstruction from Monte Carlo Data

从蒙特卡洛数据重构

An alternative avenue to obtain form factors from first principles is their reconstruction from Monte Carlo data. For the case of quantum gravity, this approach has been pioneered in [16], using correlation functions obtained within the Causal Dynamical Triangulation (CDT) program [125, 126] on a toroidal topology. The data used in the reconstruction utilized that the background geometry is flat in combination with the auto-correlation function of three-volume fluctuations,

从第一性原理获取形状因子的另一途径是从蒙特卡洛数据对其进行重构。在量子引力领域，文献 [16] 率先采用了这一方法，它使用了在环面拓扑上进行的因果动态三角剖分 (CDT) 项目中得到的关联函数 [125, 126]。重构所用数据利用了背景几何为平面这一特性，并结合了三体积涨落的自关联函数，

$$\mathfrak{V}_2\left(t', t\right) = \langle \delta V_3\left(t'\right) \delta V_3\left(t\right)\rangle . \tag{117}$$

Here t and t' denote different times of the Cauchy slicing implemented in CDT. The latter has been measured for a specific set of bare couplings appearing in the path integral over causal geometries [127]. The idea of reconstructing the effective action from numerical data is clearly generally applicable to any quantum gravity approach that produces numerical data. So far, however, this algorithm has not been applied to other approaches besides CDT.

此处 t 和 t' 代表 CDT 中柯西面切片的不同时刻。对于出现在因果几何路径积分中的一组特定裸耦合，已经测量得到了后者 [127]。从数值数据重构有效作用量这一思路，显然普遍适用于任何能产生数值数据的量子引力方法。但迄今为止，该算法除 CDT 外尚未应用于其他方法。

Following the spirit of constructing the most general scattering amplitude compatible with the parameterized quantum effective action including form factors, one can identify all terms in Γ that contribute to the correlator (117). Imposing diffeomorphism invariance, all relevant structures are contained in

遵循构造与含形状因子的参数化量子有效作用量相容的最一般散射振幅的思路，我们可以识别出 Γ 中所有对关联函数 (117) 有贡献的项。施加微分同胚不变性后，所有相关结构都包含在

$$\Gamma \simeq \frac{1}{16\pi G_N} \int d^4x\sqrt{g}\left[2\Lambda - R - \frac{1}{6} R f_{RR}(\Box) R\right]. \tag{118}$$

The flat background geometry fixes $\Lambda = 0$. Subsequently, one computes the autocorrelation function (117) from (118). In [16], it was shown that this correlator reduces to

平面背景几何确定了 $\Lambda = 0$ 。随后，我们可以从 (118) 计算出自关联函数 (117)。文献 [16] 表明，该关联函数可以化简为

$$\mathfrak{V}_2(t', t) \propto \sum_n{}' \frac{1}{\lambda_n} \phi_n^*(t') \phi_n(t), \tag{119}$$

where λ_n are the eigenvalues and ϕ_n the eigenfunctions of the temporal part of the two-point function. The fitting procedure with the data from [127] then yielded the form factor

其中 λ_n 是两点函数时间部分的本征值，ϕ_n 是其本征函数。然后结合文献 [127] 的数据进行拟合，得到了形状因子

$$f_{RR}(x) = -\frac{b^2}{x^2}, \tag{120}$$

for some real constant b . This result is notable since one generally does not expect such strong infrared nonlocalities, but rather a logarithm [44]. However, precisely this form has also been investigated in the context of cosmology to phenomenologically model dark energy [128, 129].

对应某个实常数 b 。该结果十分值得关注，因为一般来说人们并不预期会出现如此强的红外非局域性，预期形式应为对数 [44]。但恰好这种形式也已经在宇宙学背景下被研究，用于唯象建模暗能量 [128, 129]。

Conclusions

结论

The effective action Γ provides a powerful tool to analyze quantum gravity effects within a broad range of quantum gravity programs building on the principles of quantum field theory. By construction, Γ takes into account all quantum effects. Thus, quantum-corrected spacetimes may be obtained by solving the equations of motion provided by Γ [86, 130] . Furthermore, scattering processes can be analyzed using tree-level Feynman diagrams built from the effective propagators and vertices derived from Γ . These properties turn the effective action in a pivotal element connecting fundamental computations to the resulting phenomenological consequences.

有效作用量 Γ 是在诸多基于量子场论原理的量子引力研究框架中分析量子引力效应的有力工具。根据构造，Γ 已经纳入了所有量子效应，因此可以通过求解由 Γ [86, 130] 给出的运动方程得到量子修正后的时空。此外，散射过程可以利用由 Γ 导出的有效传播子和顶点构建的树级费曼图进行分析。这些性质使得有效作用量成为连接基础计算与唯象结果的核心枢纽。

Within Γ, quantum effects manifest themselves in terms of form factors. These generalize the momentum-dependent couplings found in perturbative quantum field theory to arbitrary curved spacetimes in a background-independent way. Notably, they are capable of capturing genuine non-perturbative effects including, e.g., an anomalous dimension of the graviton propagator at trans-Planckian energy, $\eta_\infty = -2$.

在 Γ 框架内，量子效应通过形状因子体现出来。形状因子将微扰量子场论中依赖动量的耦合推广到任意弯曲时空，且满足背景无关性。值得注意的是，它能够描述真正的非微扰效应，例如跨普朗克能量下引力子传播子的反常维，$\eta_\infty = -2$ 。

The form factor framework allows us to determine the most general scattering amplitudes compatible with quantum field theory. The underlying two-step process is illustrated in sections ”The Quantum Effective Action Including Form Factors” and ”Classifying Two-to-Two Scattering Processes”. It first identifies all action monomials that contribute to a scattering process with a fixed set of external fields and subsequently constructs the corresponding on-shell amplitude. Conceptually, working with these amplitudes is very appealing since they constitute physical observables that are gauge-independent, invariant with respect to field redefinitions, and depend on the essential couplings (in Weinberg’s sense [4]) only, as discussed in section ”Asymptotically Safe Scattering Amplitudes”. Moreover, their Lorentzian nature gives direct access to questions related to unitarity, causality, and the non-perturbative physics that could render gravity asymptotically safe. These features leave an imprint on the resulting low-energy effective field theory, and we exemplify some of the conjectured bounds in section ”Connection to Low-Energy Effective Field Theory”.

形状因子框架可以帮助我们确定与量子场论相容的最一般散射振幅。其 underlying 的两步过程在“包含形状因子的量子有效作用量”和“二对二散射过程分类”两节中有所说明: 第一步先确定对给定外场集合的散射过程有贡献的所有作用单项式，再构造对应的在壳振幅。从概念上看，使用这类振幅非常有吸引力，正如“渐近安全散射振幅”一节讨论的那样，它们是规范不变、场重定义不变的物理可观测量，并且仅依赖 (温伯格意义下的 [4]) 本质耦合。此外，它们的洛伦兹本性可以让我们直接研究幺正性、因果性，以及可能使引力成为渐近安全的非微扰物理。这些特征会在低能有效场论中留下印记，我们在“与低能有效场论的联系”一节举例说明了部分猜想得到的边界。

The close relation of Γ to phenomenology motivates deriving this quantity (and in particular the form factors appearing in it) from first principles. Depending on the details of the microscopic theory, one may resort to standard perturbative quantization techniques. This applies, for instance, to the case of quadratic gravity discussed in section ”Realizing Asymptotically Safe Amplitudes via Form Factors”. At the non-perturbative level, Γ can be obtained by solving the Wetterich equation [13,14] adapted to gravity [5]. In this case, the effective action appears as the end point of a renormalization group trajectory in the limit where all quantum fluctuations have been integrated out. Conceptually, the Wetterich equation then determines the couplings and form factors in Γ in terms of the relevant deformations of a renormalization group fixed point. The status of first principle computations seeking to determine the properties of form factors (foremost the graviton propagator in flat Euclidean backgrounds) along these lines is reviewed in section ”Form Factors from First Principles”. Alternatively, one could start from correlation functions measured in Monte Carlo approaches to quantum gravity to reverse-engineer the relevant terms in Γ giving rise to these correlations. In the context of Causal Dynamical Triangulations [125, 126] this strategy has been pioneered in [16]. As detailed in section ”Reconstruction from Monte Carlo Data”, this provided first clues that the form factors in the gravitational sector could contain inverse powers of the Laplacian.

> Γ 与唯象学联系紧密，这推动了从第一性原理推导该量 (尤其是其中包含的形状因子) 的研究。根据微观理论的具体细节，可以采用标准的微扰量子化技术，例如“通过形状因子实现渐近安全振幅”一节讨论的二次引力就属于这种情况。在非微扰层面，Γ 可以通过求解适配引力的 Wetterich 方程 [5][13,14] 得到。在此框架下，有效作用量是所有量子涨落被积分掉后，重整化群流轨迹的端点。从概念上讲，Wetterich 方程会根据重整化群不动点的相关形变确定 Γ 中的耦合与形状因子。“第一性原理导出形状因子”一节回顾了沿着这一路径确定形状因子性质 (主要是平坦欧几里得背景下的引力子传播子) 的第一性原理计算的研究现状。另一种方法是，可以从量子引力蒙特卡洛方法中测量得到的关联函数反推得到 Γ 中产生这些关联的相关项。在因果动态三角剖分的背景下 [125, 126]，文献 [16] 率先开展了这一策略的研究。正如“从蒙特卡洛数据重构”一节的详细介绍，该研究已经给出了引力领域形状因子可能包含拉普拉斯逆幂项的初步线索。

The present chapter illustrates that form factors play a crucial role in the gravitational Asymptotic Safety program. However, their relevance is, by no means, limited to it. In particular, they also constitute the key ingredients when formulating nonlocal, ghost-free gravity [18-21] and perturbatively super-renormalizable theories of quantum gravity [22-24]. This discussion is beyond the scope of this chapter, and the interested reader is encouraged to consult the relevant chapters in this handbook for further information.

> 本章说明形状因子在引力渐近安全项目中发挥着关键作用，但它们的适用性绝不仅限于此。尤其是，它们还是构造非局域无鬼引力 [18-21] 和微扰超可重整化量子引力理论 [22-24] 的核心要素。相关讨论超出了本章范围，感兴趣的读者可以查阅本手册的相关章节获取更多信息。

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References

参考文献

1. R. Percacci, An Introduction to Covariant Quantum Gravity and Asymptotic Safety. 100 Years of General Relativity, vol. 3 (World Scientific, 2017). https://doi.org/10.1142/10369

2. M. Reuter, F. Saueressig, Quantum Gravity and the Functional Renormalization Group (Cambridge University Press, Cambridge, UK, 2019)

3. A. Eichhorn, An asymptotically safe guide to quantum gravity and matter. Front. Astron. Space Sci. 5, 47 (2019) [1810.07615]

4. S. Weinberg, Ultraviolet divergences in quantum theories of gravitation, in ed. by S.W. Hawking, W. Israel General Relativity: An Einstein Centenary Survey, vol. 790 (Cambridge University Press, Cambridge, UK, 1979)

5. M. Reuter, Nonperturbative evolution equation for quantum gravity. Phys. Rev. D57, 971 (1998) [hep-th/9605030]

6. C. Wetterich, Quantum scale symmetry [1901.04741]

7. B. Knorr, C. Ripken, F. Saueressig, Form Factors in Asymptotic Safety: conceptual ideas and computational toolbox. Class. Quant. Grav. 36, 234001 (2019) [1907.02903]

8. X.O. Camanho, J.D. Edelstein, J. Maldacena, A. Zhiboedov, Causality constraints on corrections to the graviton three-point coupling. JHEP 02, 020 (2016) [1407.5597]

9. C. Cheung, G.N. Remmen, Infrared consistency and the weak gravity conjecture. JHEP 12, 087 (2014) [1407.7865]

10. J. Tokuda, K. Aoki, S. Hirano, Gravitational positivity bounds. JHEP 11, 054 (2020) [2007.15009]

11. L. Alberte, C. de Rham, S. Jaitly, A.J. Tolley, QED positivity bounds. Phys. Rev. D 103, 125020 (2021) [2012.05798]

12. L. Alberte, C. de Rham, S. Jaitly, A.J. Tolley, Reverse bootstrapping: IR lessons for UV physics. Phys. Rev. Lett. 128, 051602 (2022) [2111.09226]

13. C. Wetterich, Exact evolution equation for the effective potential. Phys. Lett. B301, 90 (1993)

14. T.R. Morris, The exact renormalization group and approximate solutions. Int. J. Mod. Phys. A9, 2411 (1994) [hep-ph/9308265]

15. M. Reuter, Effective average actions and nonperturbative evolution equations, in 5th Hellenic School and Workshops on Elementary Particle Physics, vol. 2, 1996 [hep-th/9602012]

16. B. Knorr, F. Saueressig, Towards reconstructing the quantum effective action of gravity. Phys. Rev. Lett. 121, 161304 (2018) [1804.03846]

17. J.F. Donoghue, M.M. Ivanov, A. Shkerin, EPFL Lectures on General Relativity as a Quantum Field Theory [1702.00319]

18. T. Biswas, A. Mazumdar, W. Siegel, Bouncing universes in string-inspired gravity. JCAP 0603, 009 (2006) [hep-th/0508194]

19. T. Biswas, E. Gerwick, T. Koivisto, A. Mazumdar, Towards singularity and ghost free theories of gravity. Phys. Rev. Lett. 108, 031101 (2012) [1110.5249]

20. L. Buoninfante, A.S. Koshelev, G. Lambiase, A. Mazumdar, Classical properties of non-local, ghost- and singularity-free gravity. JCAP 09, 034 (2018) [1802.00399]

21. L. Buoninfante, G. Lambiase, Y. Miyashita, W. Takebe, M. Yamaguchi, Generalized ghost-free propagators in nonlocal field theories. Phys. Rev. D 101, 084019 (2020) [2001.07830]

22. L. Modesto, L. Rachwal, Super-renormalizable and finite gravitational theories. Nucl. Phys. B 889, 228 (2014) [1407.8036]

23. L. Modesto, L. Rachwal, I.L. Shapiro, Renormalization group in super-renormalizable quantum gravity. Eur. Phys. J. C 78, 555 (2018) [1704.03988]

24. L. Modesto, L. Rachwal, Nonlocal quantum gravity: a review. Int. J. Mod. Phys. D26, 1730020 (2017)

25. T. Draper, B. Knorr, C. Ripken, F. Saueressig, Finite quantum gravity amplitudes: no strings attached. Phys. Rev. Lett. 125, 181301 (2020) [2007.00733]

26. T. Draper, B. Knorr, C. Ripken, F. Saueressig, Graviton-mediated scattering amplitudes from the quantum effective action. JHEP 11, 136 (2020) [2007.04396]

27. B. Knorr, C. Ripken, F. Saueressig, Form factors in quantum gravity: contrasting non-local, ghost-free gravity and asymptotic safety. Nuovo Cim C 45, 28 (2022) [2111.12365]

28. B. Knorr, S. Pirlo, C. Ripken, F. Saueressig, Cartographing gravity-mediated scattering amplitudes: scalars and photons [2205.01738]

29. M. Asorey, J.L. Lopez, I.L. Shapiro, Some remarks on high derivative quantum gravity. Int. J. Mod. Phys. A 12, 5711 (1997) [hep-th/9610006]

30. A. Codello, O. Zanusso, On the non-local heat kernel expansion. J. Math. Phys. 54, 013513 (2013) [1203.2034]

31. PARTICLE DATA GROUP collaboration, Review of particle physics. Chin. Phys. C 40, 100001 (2016)

32. A. Bonanno, A. Eichhorn, H. Gies, J.M. Pawlowski, R. Percacci, M. Reuter et al., Critical reflections on asymptotically safe gravity. Front. Phys. 8, 269 (2020) [2004.06810]

33. S.A. Fulling, R.C. King, B.G. Wybourne, C.J. Cummins, Normal forms for tensor polynomials. 1: The Riemann tensor. Class. Quant. Grav. 9, 1151 (1992)

34. Y. Decanini, A. Folacci, FKWC-bases and geometrical identities for classical and quantum field theories in curved spacetime [0805.1595]

35. S.D. Chowdhury, A. Gadde, T. Gopalka, I. Halder, L. Janagal, S. Minwalla, Classifying and constraining local four photon and four graviton S-matrices. JHEP 02, 114 (2020) [1910.14392]

36. B. Knorr, Lessons from conformally reduced quantum gravity. Class. Quant. Grav. 38, 065003 (2021) [2010.00492]

37. A. Barvinsky, G. Vilkovisky, Covariant perturbation theory. 2: Second order in the curvature. General algorithms. Nucl. Phys. B 333, 471 (1990)

38. A.O. Barvinsky, Y.V. Gusev, V.V. Zhytnikov, G.A. Vilkovisky, Covariant perturbation theory. 4. Third order in the curvature [0911.1168]

39. F.J. Wegner, Some invariance properties of the renormalization group. J. Phys. C: Solid State Phys. 7, 2098 (1974)

40. S.W. Hawking, W.E. Israel, General Relativity - An Einstein Centenary Survey (University Press, Cambridge, 1979)

41. J.A. Dietz, T.R. Morris, Redundant operators in the exact renormalisation group and in the f(R) approximation to asymptotic safety. JHEP 07, 064 (2013) [1306.1223]

42. A. Baldazzi, R.B.A. Zinati, K. Falls, Essential renormalisation group. SciPost Phys. 13, 085 (2022) [2105.11482]

43. A. Baldazzi, K. Falls, Essential quantum Einstein gravity. Universe 7, 294 (2021) [2107.00671]

44. G. 't Hooft, M.J.G. Veltman, One loop divergencies in the theory of gravitation. Annales Poincare Phys. Theor. A20, 69 (1974)

45. M.H. Goroff, A. Sagnotti, Quantum gravity at two loops. Phys. Lett. B160, 81 (1985)

46. M.H. Goroff, A. Sagnotti, The ultraviolet behavior of Einstein gravity. Nucl. Phys. B266, 709 (1986)

47. J.F. Donoghue, Leading quantum correction to the Newtonian potential. Phys. Rev. Lett. 72, 2996 (1994) [gr-qc/9310024]

48. J.F. Donoghue, General relativity as an effective field theory: the leading quantum corrections. Phys. Rev. D50, 3874 (1994) [gr-qc/9405057]

49. N.E.J. Bjerrum-Bohr, J.F. Donoghue, B.R. Holstein, L. Planté, P. Vanhove, Bending of light in quantum gravity. Phys. Rev. Lett. 114, 061301 (2015) [1410.7590]

50. N.E.J. Bjerrum-Bohr, B.R. Holstein, J.F. Donoghue, L. Planté, P. Vanhove, Illuminating light bending. PoS CORFU2016, 077 (2017) [1704.01624]

51. A.M. Polyakov, De Sitter space and eternity. Nucl. Phys. B 797, 199 (2008) [0709.2899]

52. P.R. Anderson, E. Mottola, Instability of global de Sitter space to particle creation. Phys. Rev. D 89, 104038 (2014) [1310.0030]

53. P.R. Anderson, E. Mottola, D.H. Sanders, Decay of the de Sitter Vacuum. Phys. Rev. D 97, 065016 (2018) [1712.04522]

54. T. Markkanen, A. Rajantie, Massive scalar field evolution in de Sitter. JHEP 01, 133 (2017) [1607.00334]

55. E. Witten, Quantum gravity in de Sitter space, in Strings 2001: International Conference, vol. 6, 2001 [hep-th/0106109]

56. R. Bousso, Cosmology and the S-matrix. Phys. Rev. D 71, 064024 (2005) [hep-th/0412197]

57. D. Marolf, I.A. Morrison, M. Srednicki, Perturbative S-matrix for massive scalar fields in global de Sitter space. Class. Quant. Grav. 30, 155023 (2013) [1209.6039]

58. S. Mandal, S. Banerjee, Local description of S-matrix in quantum field theory in curved spacetime using Riemann-normal coordinate. Eur. Phys. J. Plus 136, 1064 (2021) [1908.06717]

59. S.B. Giddings, The gravitational S-matrix: Erice lectures. Subnucl. Ser. 48, 93 (2013) [1105.2036]

60. B. Knorr, C. Ripken, Scattering amplitudes in affine gravity. Phys. Rev. D 103, 105019 (2021) [2012.05144]

61. R. Ferrero, C. Ripken, De Sitter scattering amplitudes in the Born approximation [2112.03766]

62. S. Weinberg, Infrared photons and gravitons. Phys. Rev. 140, B516 (1965)

63. Y.S. Jin, A. Martin, Number of subtractions in fixed-transfer dispersion relations. Phys. Rev. 135, B1375 (1964)

64. K. Stelle, Renormalization of higher derivative quantum gravity. Phys. Rev. D 16, 953 (1977)

65. J.F. Donoghue, G. Menezes, Arrow of causality and quantum gravity. Phys. Rev. Lett. 123, 171601 (2019) [1908.04170]

66. D. Anselmi, M. Piva, Quantum gravity, fakeons and microcausality. JHEP 11, 021 (2018) [1806.03605]

67. D. Anselmi, Fakeons, microcausality and the classical limit of quantum gravity. Class. Quant. Grav. 36, 065010 (2019) [1809.05037]

68. J.F. Donoghue, G. Menezes, Causality and gravity. JHEP 11, 010 (2021) [2106.05912]

69. J.F. Donoghue, G. Menezes, Massive poles in Lee-Wick quantum field theory. Phys. Rev. D 99, 065017 (2019) [1812.03603]

70. M. Reuter, F. Saueressig, Renormalization group flow of quantum gravity in the Einstein-Hilbert truncation. Phys. Rev. D65, 065016 (2002) [hep-th/0110054]

71. A. Platania, Causality, unitarity and stability in quantum gravity: a non-perturbative perspective. JHEP 09, 167 (2022) [2206.04072]

72. E. Palti, The swampland: introduction and review. Fortsch. Phys. 67, 1900037 (2019) [1903.06239]

73. I. Basile, A. Platania, Asymptotic safety: swampland or wonderland? Universe 7, 389 (2021) [2107.06897]

74. D. Becker, C. Ripken, F. Saueressig, On avoiding Ostrogradski instabilities within Asymptotic Safety. JHEP 12, 121 (2017) [1709.09098]

75. A. Platania, C. Wetterich, Non-perturbative unitarity and fictitious ghosts in quantum gravity. Phys. Lett. B 811, 135911 (2020) [2009.06637]

76. T.N. Pham, T.N. Truong, Evaluation of the derivative quartic terms of the meson chiral Lagrangian from forward dispersion relation. Phys. Rev. D 31, 3027 (1985)

77. B. Ananthanarayan, D. Toublan, G. Wanders, Consistency of the chiral pion pion scattering amplitudes with axiomatic constraints. Phys. Rev. D 51, 1093 (1995) [hep-ph/9410302]

78. A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis, R. Rattazzi, Causality, analyticity and an IR obstruction to UV completion. JHEP 10, 014 (2006) [hep-th/0602178]

79. U. Harst, M. Reuter, QED coupled to QEG. JHEP 1105, 119 (2011) [1101.6007]

80. G.P. de Brito, A. Eichhorn, M. Schiffer, Light charged fermions in quantum gravity. Phys. Lett. B 815, 136128 (2021) [2010.00605]

81. H. Gies, J. Ziebell, Asymptotically safe QED. Eur. Phys. J. C 80, 607 (2020) [2005.07586]

82. N. Arkani-Hamed, L. Motl, A. Nicolis, C. Vafa, The String landscape, black holes and gravity as the weakest force. JHEP 06, 060 (2007) [hep-th/0601001]

83. I.T. Drummond, S.J. Hathrell, QED vacuum polarization in a background gravitational field and its effect on the velocity of photons. Phys. Rev. D 22, 343 (1980)

84. A. Eichhorn, Quantum-gravity-induced matter self-interactions in the asymptotic-safety scenario. Phys. Rev. D86, 105021 (2012) [1204.0965]

85. C. Laporte, A.D. Pereira, F. Saueressig, J. Wang, Scalar-tensor theories within Asymptotic Safety. JHEP 12, 001 (2021) [2110.09566]

86. D. Becker, M. Reuter, Towards a C -function in 4D quantum gravity. JHEP 03,065 (2015) [1412.0468]

87. J.D. Edelstein, R. Ghosh, A. Laddha, S. Sarkar, Causality constraints in quadratic gravity. JHEP 09, 150 (2021) [2107.07424]

88. Z. Bern, D. Kosmopoulos, A. Zhiboedov, Gravitational effective field theory islands, low-spin dominance, and the four-graviton amplitude. J. Phys. A 54, 344002 (2021) [2103.12728]

89. M. Niedermaier, M. Reuter, The asymptotic safety scenario in quantum gravity. Living Rev. Rel. 9, 5 (2006)

90. A. Codello, R. Percacci, C. Rahmede, Investigating the ultraviolet properties of gravity with a Wilsonian renormalization group equation. Ann. Phys. 324, 414 (2009) [0805.2909]

91. M. Reuter, F. Saueressig, Quantum Einstein gravity. New J.Phys. 14, 055022 (2012) [1202.2274]

92. N. Dupuis, L. Canet, A. Eichhorn, W. Metzner, J.M. Pawlowski, M. Tissier et al., The nonperturbative functional renormalization group and its applications. Phys. Rep. (2020) [2006.04853]

93. F. Saueressig, The Functional Renormalization Group in Quantum Gravity. arXiv:2302.14152

94. J.M. Pawlowski, M. Reichert, Quantum gravity: a fluctuating Point of view. Front. Phys. 8, 551848 (2021) [2007.10353]

95. E.V. Gorbar, I.L. Shapiro, Renormalization group and decoupling in curved space. JHEP 02, 021 (2003) [hep-ph/0210388]

96. E.V. Gorbar, I.L. Shapiro, Renormalization group and decoupling in curved space. 2. The Standard model and beyond JHEP 06, 004 (2003) [hep-ph/0303124]

97. Y. Decanini, A. Folacci, Off-diagonal coefficients of the DeWitt-Schwinger and Hadamard representations of the Feynman propagator. Phys. Rev. D73, 044027 (2006) [gr-qc/0511115]

98. D. Benedetti, K. Groh, P.F. Machado, F. Saueressig, The universal RG machine. JHEP 1106, 079 (2011) [1012.3081]

99. A. Codello, Polyakov effective action from functional renormalization group equation. Ann. Phys. 325, 1727 (2010) [1004.2171]

100. A. Satz, A. Codello, F. Mazzitelli, Low energy quantum gravity from the effective average action. Phys. Rev. D 82, 084011 (2010) [1006.3808]

101. A. Codello, R. Percacci, L. Rachwal, A. Tonero, Computing the effective action with the functional renormalization group. Eur. Phys. J. C 76, 226 (2016) [1505.03119]

102. A. Codello, R.K. Jain, On the covariant formalism of the effective field theory of gravity and leading order corrections. Class. Quant. Grav. 33, 225006 (2016) [1507.06308]

103. N. Ohta, L. Rachwal, Effective action from the functional renormalization group. Eur. Phys. J. C 80, 877 (2020) [2002.10839]

104. B. Knorr, M. Schiffer, Non-perturbative propagators in quantum gravity. Universe 7, 216 (2021) [2105.04566]

105. A. Codello, R.K. Jain, On the covariant formalism of the effective field theory of gravity and its cosmological implications. Class. Quant. Grav. 34, 035015 (2017) [1507.07829]

106. L. Bosma, B. Knorr, F. Saueressig, Resolving spacetime singularities within asymptotic safety. Phys. Rev. Lett. 123, 101301 (2019) [1904.04845]

107. A. Bonanno, M. Reuter, Renormalization group improved black hole space-times. Phys. Rev. D62, 043008 (2000) [hep-th/0002196]

108. J. Meibohm, J.M. Pawlowski, M. Reichert, Asymptotic safety of gravity-matter systems. Phys. Rev. D93, 084035 (2016) [1510.07018]

109. E. Manrique, M. Reuter, Bimetric truncations for quantum Einstein gravity and asymptotic safety. Ann. Phys. 325, 785 (2010) [0907.2617]

110. E. Manrique, M. Reuter, F. Saueressig, Bimetric renormalization group flows in quantum Einstein gravity. Ann. Phys. 326, 463 (2011) [1006.0099]

111. N. Christiansen, D.F. Litim, J.M. Pawlowski, A. Rodigast, Fixed points and infrared completion of quantum gravity. Phys. Lett. B728, 114 (2014) [1209.4038]

112. A. Codello, G. D'Odorico, C. Pagani, Consistent closure of renormalization group flow equations in quantum gravity. Phys. Rev. D89, 081701 (2014) [1304.4777]

113. N. Christiansen, B. Knorr, J.M. Pawlowski, A. Rodigast, Global flows in quantum gravity. Phys. Rev. D93, 044036 (2016) [1403.1232]

114. N. Christiansen, B. Knorr, J. Meibohm, J.M. Pawlowski, M. Reichert, Local quantum gravity. Phys. Rev. D92, 121501 (2015) [1506.07016]

115. T. Denz, J.M. Pawlowski, M. Reichert, Towards apparent convergence in asymptotically safe quantum gravity. Eur. Phys. J. C78, 336 (2018) [1612.07315]

116. N. Christiansen, K. Falls, J.M. Pawlowski, M. Reichert, Curvature dependence of quantum gravity. Phys. Rev. D97, 046007 (2018) [1711.09259]

117. A. Bonanno, T. Denz, J.M. Pawlowski, M. Reichert, Reconstructing the graviton. SciPost Phys. 12, 001 (2022) [2102.02217]

118. J. Fehre, D.F. Litim, J.M. Pawlowski, M. Reichert, Lorentzian quantum gravity and the graviton spectral function [2111.13232]

119. S. Folkerts, D.F. Litim, J.M. Pawlowski, Asymptotic freedom of Yang-Mills theory with gravity. Phys. Lett. B709, 234 (2012) [1101.5552]

120. N. Christiansen, D.F. Litim, J.M. Pawlowski, M. Reichert, Asymptotic safety of gravity with matter. Phys. Rev. D97, 106012 (2018) [1710.04669]

121. A. Eichhorn, P. Labus, J.M. Pawlowski, M. Reichert, Effective universality in quantum gravity. SciPost Phys. 5, 031 (2018) [1804.00012]

122. A. Eichhorn, S. Lippoldt, J.M. Pawlowski, M. Reichert, M. Schiffer, How perturbative is quantum gravity? Phys. Lett. B792, 310 (2019) [1810.02828]

123. A. Eichhorn, S. Lippoldt, M. Schiffer, Zooming in on fermions and quantum gravity. Phys. Rev. D99, 086002 (2019) [1812.08782]

124. B. Bürger, J.M. Pawlowski, M. Reichert, B.-J. Schaefer, Curvature dependence of quantum gravity with scalars [1912.01624]

125. J. Ambjørn, A. Görlich, J. Jurkiewicz, R. Loll, Nonperturbative quantum gravity. Phys. Rept. 519, 127 (2012) [1203.3591]

126. R. Loll, Quantum gravity from causal dynamical triangulations: a review. Class. Quant. Grav. 37, 013002 (2020) [1905.08669]

127. J. Ambjørn, Z. Drogosz, J. Gizbert-Studnicki, A. Görlich, J. Jurkiewicz, D. Nemeth, Impact of topology in causal dynamical triangulations quantum gravity. Phys. Rev. D94, 044010 (2016) [1604.08786]

128. M. Maggiore, M. Mancarella, Nonlocal gravity and dark energy. Phys. Rev. D90, 023005 (2014) [1402.0448]

129. E. Belgacem, Y. Dirian, S. Foffa, M. Maggiore, Nonlocal gravity. Conceptual aspects and cosmological predictions. JCAP 1803, 002 (2018) [1712.07066]

130. B. Knorr, A. Platania, Sifting quantum black holes through the principle of least action. Phys. Rev. D 106, L021901 (2022) [2202.01216]